Problem Solving with Rational Functions   March 28/2019

Ex.1. A consultant has issued an environmental report on the cost of cleaning up a property that was previously the site of a chemical factory. Costs can increase dramatically depending on the percent of pollutants that needs to be removed.

Her report gives the cost, C, in dollars, of removing p% of the pollutants from the site as:

$$C(p) = \frac{50000}{100 - p}$$

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a) What is the cost of removal for half of the pollutants? $C(p) = \frac{50000}{100 - p}$

$$p = 50 \qquad C(50) = \frac{50000}{100 - 50}$$

$$= 1000$$

∴ it would cost $1000.

b) What is the cost of removal for 90% of the pollutants?
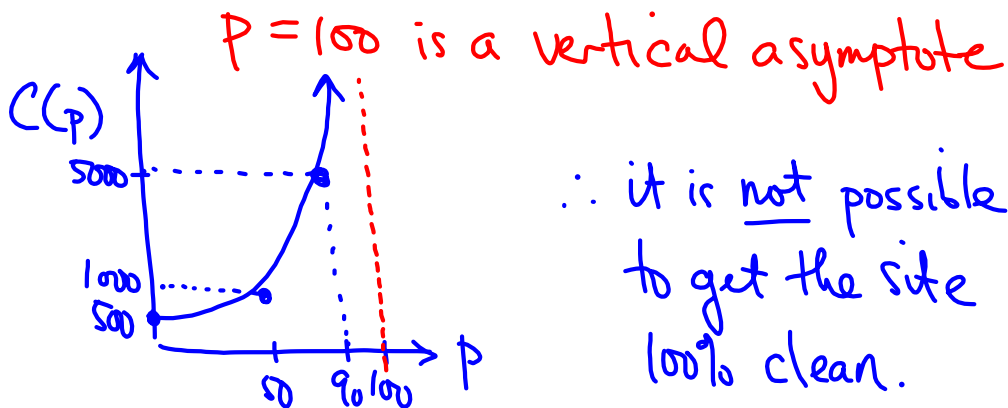
$$C(90) = \frac{50000}{100 - 90}$$

$$= 5000$$

∴ it would cost $5000.

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c) Would it be affordable to remove all of the pollutants? $C(p) = \dfrac{50000}{100 - p}$

$$C(100) = \frac{50000}{100 - 100}$$

$$= \frac{50000}{0} \quad \text{undefined}$$

P = 100 is a vertical asymptote

$C(p)$

∴ it is **not** possible to get the site 100% clean.

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Ex.2 Fred drove his car a distance of 2x km in 3 hours. Later, he drove a distance of (x + 100) km in 2 hours.

a) Write an expression for the first speed.   Use the equation:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$v_1 = \frac{2x}{3}$$

b) Write an expression for the second speed.

$$v_2 = \frac{x + 100}{2}$$

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Ex.2 Fred drove his car a distance of 2x km in 3 hours. Later, he drove a distance of (x + 100) km in 2 hours.

$$speed = \frac{distance}{time}$$

c) Write a <u>simplified</u> expression for the difference
    between the first speed and the second speed.

$$v_1 - v_2 = \frac{2x}{3} - \frac{x+100}{2}$$

$$= \frac{2x(2) - (x+100)(3)}{6}$$

$$= \frac{(4x) - (3x+300)}{6}$$

$$= \frac{x-300}{6}$$

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Ex.2 Fred drove his car a distance of 2x km in 3 hours. Later, he drove a distance of (x + 100) km in 2 hours.

$$speed = \frac{distance}{time}$$

d) What is the distance where the first and second
    speed are the same?

$$v_1 - v_2 = \frac{x-300}{6}$$

for $v_1 = v_2$, $v_1 - v_2 = 0$

$$\frac{(x-300)}{6} = 0 \qquad [\times 6]$$

$$x - 300 = 0$$

$$x = 300$$

$$d_1 = 2x \qquad d_2 = x+100$$
$$= 2(300) \qquad = 300 + 100$$
$$= 600 \qquad = 400$$

∴ the first trip is 600km and
   the second is 400 km when the
   speeds are the same.

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Ex.3 A small plane flies along a straight path from Ottawa to Toronto and back again. The straight-line distance between these cities is 480 km.

(a) On Monday, there was a constant wind blowing from Toronto to Montreal at 60 km/h. The round-trip took 3 hours. What was the airspeed of the plane?

$$O \xrightarrow{\quad v \quad} T$$

Ottawa 60

$$O \xleftarrow{\quad} T$$
$$60 \text{km/h}$$

let v represent the airspeed (km/h)

$O \xleftarrow{\quad} T$

$d_1 = d_2 = 480 \text{km}$

$v_1 = v - 60$ (to T) $v_2 = v + 60$ (to O)

$t_1 + t_2 = 3$

$\dfrac{d_1}{v_1} + \dfrac{d_2}{v_2} = 3$

$\dfrac{480}{v-60} + \dfrac{480}{v+60} = 3$

$v = \dfrac{d}{t}$

$d = vt$

$t = \dfrac{d}{v}$ ✱

LCD: $(v-60)(v+60)$

$\dfrac{480(v+60) + 480(v-60)}{(v-60)(v+60)} = 3$

$\dfrac{(480v + 28800) + (480v - 28800)}{(v-60)(v+60)} = 3$

$\dfrac{960v}{(v-60)(v+60)} = 3$

$960v = 3(v-60)(v+60)$

$960v = 3(v^2 - 3600)$

$960v = 3v^2 - 10800$

$0 = 3v^2 - 960v - 10800$

$0 = \underset{a}{v^2} - \underset{b}{320v} - \underset{c}{3600}$

$v = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$v = \dfrac{-(-320) \pm \sqrt{(-320)^2 - 4(1)(-3600)}}{2(1)}$

$v = \dfrac{320 \pm \sqrt{116800}}{2}$

$v \doteq 330.9$ OR $v = -10.9$
$\qquad\qquad\qquad$ Speed cannot be negative

∴ the airspeed is 330.9 km/h.

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Ex.3 A small plane flies along a straight path from Ottawa to Toronto andback again. The straight-line distance between these cities is 480 km.

(b) On Friday, there was no wind. How long was the round-trip?
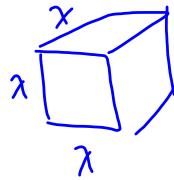
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Assigned Work:

  p.69 #12, 13
  p.51 #10
  p.42 #11

  + problems on worksheet   **3 , 4**

p.42 #11.   ratio : $\dfrac{\text{Volume}}{\text{Surface area}}$

$$V = x^3$$

$$SA = 6x^2$$

$$\text{ratio} = \frac{V}{SA}$$

$$= \frac{x^3}{6x^2}$$

$$= \frac{x}{6} , x \neq 0$$

$$\frac{x \cdot \cancel{x} \cdot \cancel{x}^{1}}{6 \cdot \cancel{x}_{1} \cdot \cancel{x}_{1}}$$

$$= \frac{x}{6}$$

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WS #3.

   $\$239.25$

   let x represent # of students

   $x - 4$ students paid for lunch.

if all students paid	only $x-4$ paid
$\dfrac{\text{cost}}{\text{per student}} = \dfrac{239.25}{x}$	$\dfrac{239.25}{x-4}$

$$\frac{239.25}{x} + 1 = \frac{239.25}{x-4}$$

$$1 = \frac{239.25}{x-4} - \frac{239.25}{x}$$

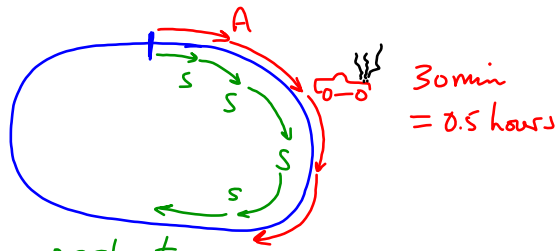$$1 = \frac{239.25x - 239.25(x-4)}{(x-4)(x)}$$

$$1 = \frac{957}{(x-4)(x)}$$

$$x(x-4) = 957$$

$$x^2 - 4x = 957$$

$$x^2 - 4x - 957 = 0$$

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WS#4

A

S   S

S

S

S

30 min
= 0.5 hours

$t_A - 0.25h = t_S$

$t_A - 15 \min = t_S$ or $t_A = t_S + 15$

$d_A = d_S = 42 km$ $v_A = v_S + 0.4 km/h$

$v = v + 0.4$

$d = vt$ $v = \dfrac{d}{t}$ $t = \dfrac{d}{v}$

$t_A - 0.25 = t_S$

$\left(\dfrac{42}{v} + 0.5\right) - 0.25 = \dfrac{42}{v + 0.4}$

total time
for A on track

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