

Recall: Exponent Laws (same base)

$$(a^x)(a^y) = a^{x+y} \qquad a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}, \quad a \neq 0$$

$$a^{-x} = \frac{1}{a^x}, \quad a \neq 0 \qquad (a^x)^y = a^{xy}$$

$$a^0 = 1, \quad a \neq 0$$

Apr 6-9:15 PM

Recall: Exponent Laws (different base)

$$(ab)^x = (a^x)(b^x)$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}, \quad b \neq 0$$

Apr 6-9:11 PM

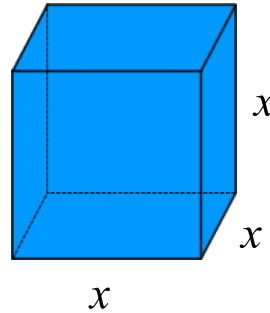
Rational Exponents

Apr. 4/2019

For a cube of side length x ,

volume: $V(x) = x^3$

area of a side: $A_{side}(x) = x^2$



How could we represent the side length, x , as a power (i.e., exponential relation) of the volume or area?

Apr 11-10:31 PM

$$x = V^n$$

Start with: $V = x^3$

$$V = x^3$$

$$V = (V^n)^3$$

$$V^1 = V^{3n}$$

$$\Rightarrow 1 = 3n$$

implies $n = \frac{1}{3}$

$$\therefore x = V^{\frac{1}{3}}$$

or

$$x = \sqrt[3]{V}$$

$$x = A^m$$

$$A = x^2$$

$$A = (A^m)^2$$

$$A^1 = A^{2m}$$

$$\Rightarrow 1 = 2m$$

$$m = \frac{1}{2}$$

$$\therefore x = A^{\frac{1}{2}}$$

or

$$x = \sqrt{A}$$

Apr 11-10:40 PM

Summary: A rational exponent is equivalent to a radical.

$$(b)^{\frac{1}{n}} = \sqrt[n]{b}, \text{ also called the } \underline{\text{nth root of } b},$$

where $n > 0$, n is an integer.

Note: Typical rules/restrictions apply, such as

- cannot divide by zero
- cannot take the even root of a negative

Ex.1 Express in radical notation, then evaluate.

(a) $10000^{\frac{1}{4}}$ (b) $(-8)^{\frac{1}{3}}$ (c) $49^{-\frac{1}{2}}$

Apr 11-10:42 PM

Ex.1 Express in radical notation, then evaluate.

(a) $10000^{\frac{1}{4}}$ (b) $(-8)^{\frac{1}{3}}$ (c) $49^{-\frac{1}{2}}$

$$\begin{aligned}
 &= \sqrt[4]{10000} & &= \sqrt[3]{-8} & &= \frac{1}{49^{\frac{1}{2}}} \\
 &= 10 & &= -2 & &= \frac{1}{\sqrt{49}} \\
 & & & & &= \frac{1}{7}
 \end{aligned}$$

Apr 11-10:56 PM

Ex.2 Express in radical notation, then evaluate.

$$(a) \quad 27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2$$

$$= \left(\sqrt[3]{27}\right)^2$$

$$= (3)^2$$

$$= 9$$

$$\frac{2}{3} = \frac{1}{3} \cdot \frac{2}{1}$$

OR

$$= \frac{2}{1} \cdot \frac{1}{3}$$

OR

$$27^{\frac{2}{3}} = \left(27^2\right)^{\frac{1}{3}}$$

$$= (729)^{\frac{1}{3}}$$

$$= \sqrt[3]{729}$$

$$= 9$$

Apr 11-10:59 PM

Ex.2 Express in radical notation, then evaluate.

$$(b) \quad (-27)^{\frac{4}{3}} = \left[(-27)^{\frac{1}{3}}\right]^4$$

$$= \left[\sqrt[3]{-27}\right]^4$$

$$= [-3]^4$$

$$= 81$$

Apr 11-11:01 PM

Summary: The numerator of a rational exponent may be a value other than 1. Use exponent laws to express as a combination of a power and a radical.

$$(b)^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$$

where n and m are integers, $n > 0$, ~~n > 0~~

Apr 11-11:02 PM

Even & Odd Roots

Recall: It is not possible to take the square root of a negative number. Why?

No number multiplied by itself will be negative.

Similarly, you cannot take any even root (2, 4, 6, etc) of a negative number.

With odd roots (3, 5, 7, etc), it is possible to have positive or negative values under the root.

Apr 11-11:08 PM

Assigned Work:

handout # (1-6)(odd), 8, 10, 12, 14

bae a a
 f
 3ceb

$$\begin{aligned}
 3(c) \quad & (-11)^2 (-11)^{\frac{3}{4}} && 2 + \frac{3}{4} \\
 & = (-11)^{2 + \frac{3}{4}} && = \frac{8}{4} + \frac{3}{4} \\
 & = (-11)^{\frac{11}{4}} && = \frac{11}{4}
 \end{aligned}$$

$$\begin{aligned}
 (e) \quad & \frac{9^{-\frac{1}{5}}}{9^{\frac{2}{3}}} = 9^{-\frac{1}{5} - (\frac{2}{3})} && -\frac{1}{5} - \frac{2}{3} \\
 & = 9^{-\frac{13}{15}} && = \frac{-3-10}{15} \\
 & && = -\frac{13}{15}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & 8^{\frac{2}{3}} \div 8^{\frac{1}{3}} = 8^{\frac{2}{3} - \frac{1}{3}} \\
 & = 8^{\frac{1}{3}}
 \end{aligned}$$

Apr 11-11:08 PM

bae

$$\begin{aligned}
 (a) \quad & 4^{\frac{1}{5}} (4^{0.3}) = 4^{\frac{1}{5} + 0.3} \\
 & = 4^{\frac{1}{5} + \frac{3}{10}} && \frac{2 \times 1}{2 \times 5} = \frac{2}{10} \\
 & = 4^{\frac{2}{10} + \frac{3}{10}} \\
 & = 4^{\frac{5}{10}} \\
 & = 4^{\frac{1}{2}} \\
 & = \sqrt{4} \\
 & = 2
 \end{aligned}$$

$$\begin{aligned}
 (e) \quad & \frac{(16^{-2.5})^{-0.2}}{16^{\frac{3}{4}}} = \frac{(16^{-\frac{5}{2}})^{-\frac{2}{10}}}{16^{\frac{3}{4}}} \\
 & = \frac{16^{(-\frac{5}{2} \times -\frac{2}{10})}}{16^{\frac{3}{4}}} \\
 & = \frac{16^{\frac{1}{2}}}{16^{\frac{3}{4}}} \\
 & = 16^{\frac{2}{4} - \frac{3}{4}} \\
 & = 16^{-\frac{1}{4}} \\
 & = \frac{1}{16^{\frac{1}{4}}} \\
 & = \frac{1}{\sqrt[4]{16}} \\
 & = \frac{1}{2}
 \end{aligned}$$

$\frac{1}{2} - \frac{3}{4}$
 $= \frac{2}{4} - \frac{3}{4}$
 $= -\frac{1}{4}$

Apr 5-12:43 PM

$$12(a) \quad -(256)^{0.375} = -8$$

14 a f

$$(a) \quad 9^{\frac{1}{2}} + 4^{\frac{1}{2}} = (9+4)^{\frac{1}{2}} ?$$

$$LS = 9^{\frac{1}{2}} + 4^{\frac{1}{2}} \quad RS = (9+4)^{\frac{1}{2}}$$

$$= \sqrt{9} + \sqrt{4} \quad = (13)^{\frac{1}{2}}$$

$$= 3 + 2 \quad = \sqrt{13}$$

$$= 5 \quad \neq 5$$

$$LS \neq RS$$

$$(f) \quad \left[(x^{\frac{1}{3}})(y^{\frac{1}{3}}) \right]^6 = x^2 y^2 ?$$

$$LS = \left[(xy)^{\frac{1}{3}} \right]^6 \quad (a^{\frac{1}{3}})^6$$

$$= (xy)^2 \quad = a^2$$

$$= x^2 y^2 \quad LS = RS$$

$$= RS \quad \therefore \text{true.}$$

Apr 5-12:49 PM