Unit 4 - Exponential Functions

Review of Exponent Laws

Exercises: p.9 #1-9(odd), 12, 16

Recall:

A **power** is a product of identical factors and consists of two parts: a **base** and an **exponent**.

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

base = the identical factor

exponent = how many factors there are altogether

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Ex. Evaluate.

a) 3^3

$= 3 \cdot 3 \cdot 3$

$= 27$

b) $(-3)^3 = (-3)(-3)(-3)$

$= -27$

c) -2^4 base = 2

$= -(2)^4$

$= -(2)(2)(2)(2)$

$= -16$

d) $\left(\frac{4}{3}\right)^2 = \left(\frac{4}{3}\right)\left(\frac{4}{3}\right)$

$= \frac{16}{9}$

or

$= \frac{4^2}{3^2}$

$= \frac{16}{9}$

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Rule #1: Multiplication of Powers with the same base

To investigate the rule let us look at a specific example and go through the process of expanding before simplifying.

$$\left(3^{1}\right)\left(3^{2}\right)=(3)(3)(3)$$
$$=27$$
$$=3^{3}$$

The Rule: $(a^{x})(a^{y})=a^{x+y}$

In words: when multiplying powers with the same base, add exponents.

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Rule #2: Division of Powers with the same base

To investigate the rule let us look at a specific example and go through the process of expanding before simplifying.

$$3^{2}\div 3^{1}=\frac{(3)(\not{3})^{1}}{\not{3}_{1}}$$
$$=3^{1}$$
$$=3^{2-1}$$

The Rule: $a^{x}\div a^{y}=\dfrac{a^{x}}{a^{y}}$

$$=a^{x-y},\ a\neq 0$$

In words: when dividing powers with the same base, subtract exponents.

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Rule #3: Power of a Power

To investigate the rule let us look at a specific example and go through the process of expanding before simplifying.

$$\left(3^2\right)^4 = (3^2)(3^2)(3^2)(3^2)$$
$$= 3^8$$
$$= 3^{(2\cdot 4)}$$

The Rule: $\left(a^x\right)^y = a^{xy}$

In words: when having a power to an exponent, multiply the exponents.

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Ex.1 Simplify. Express your final answer with a single base and exponent.

(a) $\left(4^{-6}\right)\left(4^4\right)$

$= 4^{-6+4}$

$= 4^{-2}$

(b) $\dfrac{(-3)^2}{(-3)^{-3}}$ $= (-3)^{2-(-3)}$

$= (-3)^5$

(c) $\left(5^{-2}\times 5^4\right)^{-2}$

$= \left(5^{-2+4}\right)^{-2}$

$= \left(5^2\right)^{-2}$

$= 5^{-4}$

(d) $\left(3a^2b\right)\left(-2a^3b^4\right)$

$= (3)(a^2)(b^1)(-2)(a^3)(b^4)$

commutative

$2\cdot 3\cdot 4 = 4\cdot 2\cdot 3$

$= -6a^5b^5$

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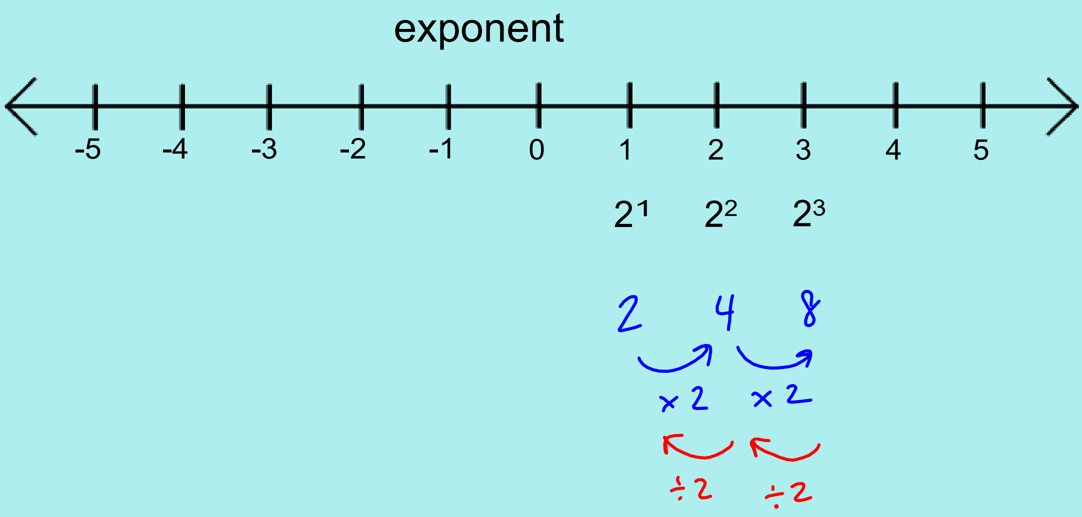
Rule #4: Identity Rule

What exponent does not change the value of a power?

The Rule: $a^1 = a$

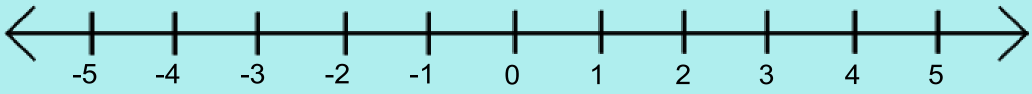
In words: anything to the exponent of 1 is equal to itself.

Patterns in Powers of 2



Patterns in Powers of 2

exponent



2^0 2^1 2^2 2^3

1 2 4 8

$\div 2$

5^0 5^1 5^2 5^3

1 5 25 125

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Rule #5: Zero Exponent

Lets look at the expanded form of powers and find a pattern:

$2^3 =$ 8

$2^2 =$ 4

$2^1 =$ 2

then $2^0 =$ 1

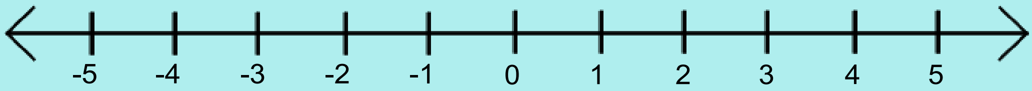
0^0 undefined

The Rule: $a^0 = 1, \ a \neq 0$

In words: anything to the exponent of zero is 1.
This is because an exponent of zero means you are dividing the base by itself.

Patterns in Powers of 2

exponent



2^{-3} 2^{-2} 2^{-1} 2^{0} 2^{1} 2^{2} 2^{3}

$2^{-3} = \frac{1}{2^3}$

~~0.5~~ 1 2 4 8

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{2}$

$\frac{1}{2^3}$ $\frac{1}{2^2}$ $\frac{1}{2^1}$

$\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

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Rule #6: Negative Exponent

Now continue the pattern from the previous rule to determine the effect of a negative in the exponent:

$2^0 = 1$

$2^{-1} = \frac{1}{2} = \frac{1}{2^1}$

$2^{-2} = \frac{1}{4} = \frac{1}{2^2}$

$= \frac{1}{a^x}$

The Rule: $a^{-x} = \left(\frac{1}{a}\right)^x, \; a \neq 0$

In words: a negative exponent requires you to find the reciprocal of the base.

Rule #7: Distributive Rule (for powers with different bases)

To investigate the rule let us look at a specific example and go through the process of expanding before simplifying.

$(7^2 \cdot 2^5)^3 = (7^2 \cdot 2^5)(7^2 \cdot 2^5)(7^2 \cdot 2^5)$

$= 7^6 \cdot 2^{15}$

$= (7^2)^3 \cdot (2^5)^3$

$\left(\frac{7^2}{2^5}\right)^3 = \frac{(7^2)^3}{(2^5)^3}$

$= \frac{7^6}{2^{15}}$

The Rules:

(a) $(ab)^x = (a^x)(b^x)$

(b) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x},\ b \neq 0$

(c) $(a^m b^n)^p = (a^m)^p \cdot (b^n)^p$

$= a^{mp} \cdot b^{np}$

(d) $\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}$

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Ex.2 Simplify. Express your final answer with a single base and positive exponent.

(a) $(4^{-6})(4^4)$

$= 4^{-2}$

$= \frac{1}{4^2}$

(b) $(5^{-2} \times 5^4)^{-2} = (5^2)^{-2}$

$= 5^{-4}$

$= \left(\frac{1}{5}\right)^4$

$= \frac{1}{5^4}$

(c) $(x^{-3}y^5)^{-3}$

$= (x^{-3})^{-3}(y^5)^{-3}$

$= x^9 y^{-15}$

$= x^9 \cdot \frac{1}{y^{15}}$

$= \frac{x^9}{y^{15}}$

(d) $(-a^5 b)^{-2}(-ab^{-2})^2$

$= (-1)^{-2}(a^5)^{-2}(b)^{-2}(-1)^2(a)^2(b^{-2})^2$

$= (-1)^0(a^{-10})(b^{-2})(a^2)(b^{-4})$

$= (1)(a^{-8})(b^{-6})$

$= \frac{1}{a^8 b^6}$

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The exponent laws also work if you have polynomials instead of numbers as exponents.

Ex.3 Simplify

(a) $\left(x^3\right)^{2a+4}$

$= x^{3(2a+4)}$

$= x^{(6a+12)}$

(b) $\left(x^{a+5}\right)\left(x^{3a+1}\right)$

$= x^{(a+5)+(3a+1)}$

$= x^{4a+6}$

(c) $\left(x^{4m-3n}\right)\div\left(x^{m+5n}\right)$

$= x^{(4m-3n)-(m+5n)}$

$= x^{3m-8n}$

(d) $x^y\left(x^{y+1}\right)^{y+2}\left(\frac{1}{x}\right)^{6y}$

$= x^y \, x^{(y+1)(y+2)} \, x^{-6y}$

$= x^{y+(y^2+3y+2)-6y}$

$= x^{(y^2-2y+2)}$

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The exponent laws:
(for powers with the same base)

$$(a^x)(a^y) = a^{x+y}$$

$$a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}, \quad a \neq 0$$

$$a^{-x} = \frac{1}{a^x}, \quad a \neq 0$$

$$\left(a^x\right)^y = a^{xy}$$

$$a^0 = 1, \quad a \neq 0$$

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The exponent laws:
(for powers with different bases)

$$(ab)^x = (a^x)(b^x)$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x},\ b \neq 0$$

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Assigned Work:

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