Graphing Exponential Functions

Part A: Compare the graphs of $y = 2x$

$y = 2x$

$y = x^2$

$y = 2^x$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$$

Table of Values:

$y = x^2$

x	y	Δy	$\Delta^2 y$
-2	4	$\times$	$\times$
-1	1	-3	$\times$
0	0	-1	2
1	1	1	2
2	4	3	2
3	9	5	2

$y = 2^x$

x	y	Δy	ratio
-2	0.25		
-1	0.5	0.25	$\frac{0.5}{0.25} = 2$
0	1	0.5	$1 \div 0.5 = 2$
1	2	1	$2 \div 1 = 2$
2	4	2	$4 \div 2 = 2$
3	8	4	$8 \div 4 = 2$

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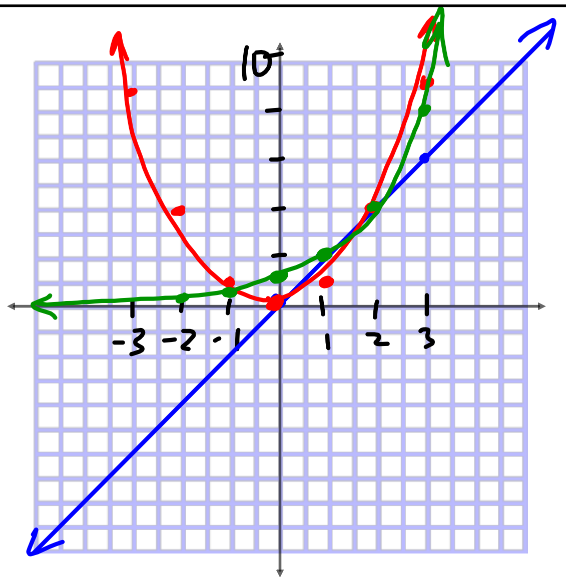
Graph $y = 2x$ ✓

$y = x^2$ ✓

$y = 2^x$ ✓

*use a different colour for each



Properties of $y = 2^x$

horizontal asymptote at $y = 0$

always increasing

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Part B: TOV and graph: 1. $y=3^x$

2. $y=4^x$

$y=3^x$

x	y	Δy	ratio
-2	$\frac{1}{9}$		
-1	$\frac{1}{3}$	$\frac{2}{9}$	3
0	1	$\frac{2}{3}$	3
1	3	2	3
2	9	6	3
3	27	18	3

$y=4^x$

x	y	ratio
-2	$\frac{1}{16}$	
-1	$\frac{1}{4}$	4
0	1	4
1	4	4
2	16	4
3	64	4

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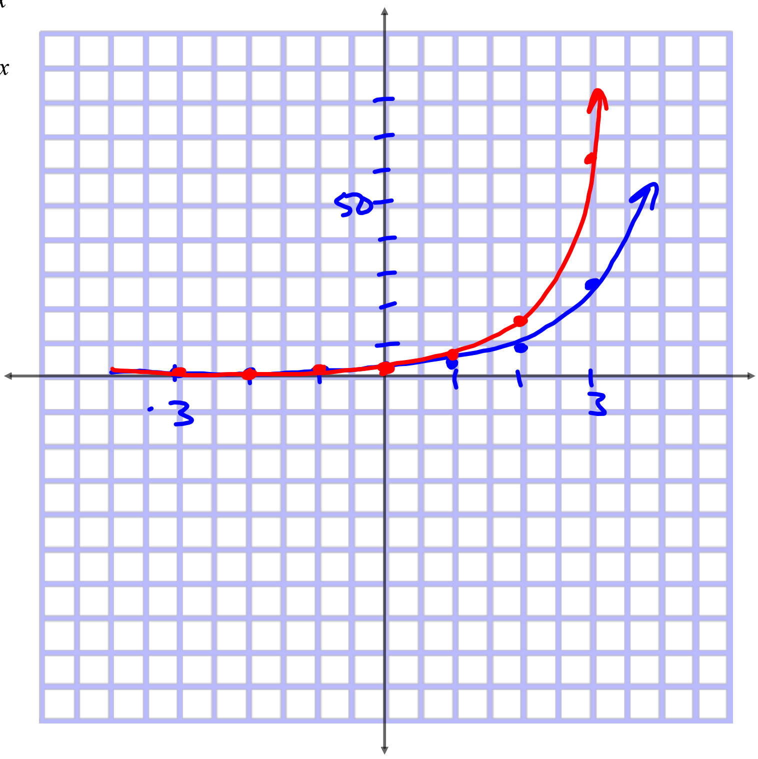
Graph 1. $y=3^x$

2. $y=4^x$

$\frac{64}{10}=6.4$



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Part B: TOV and graph:

3. $y=\left(\frac{1}{2}\right)^x$

4. $y=\left(\frac{1}{3}\right)^x$

$y=\left(\frac{1}{2}\right)^x$ $y=\left(\frac{1}{3}\right)^x$

x	y
-2	$\left(\frac{1}{2}\right)^{-2}=\left(\frac{2}{1}\right)^2=4$
-1	$\left(\frac{1}{2}\right)^{-1}=2$
0	$\left(\frac{1}{2}\right)^0=1$
1	$\left(\frac{1}{2}\right)^1=\frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2=\frac{1}{4}$

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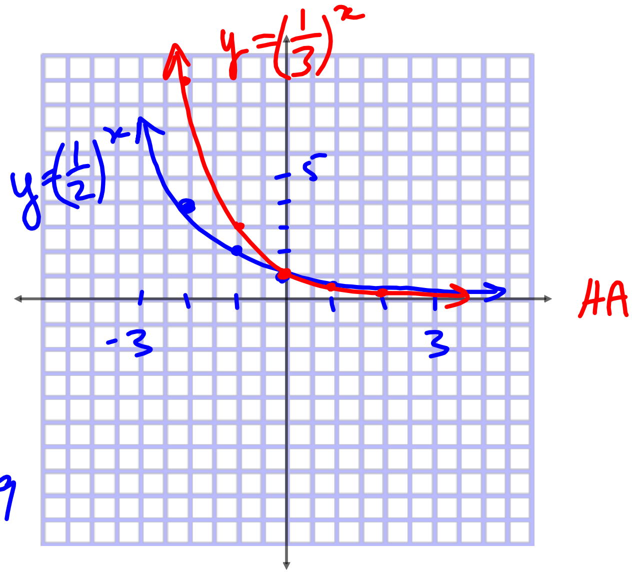
Graph 3. $y=\left(\frac{1}{2}\right)^x$

4. $y=\left(\frac{1}{3}\right)^x$



x	$y=\left(\frac{1}{3}\right)^x$
-2	$\left(\frac{1}{3}\right)^{-2}=\left(\frac{3}{1}\right)^2=9$
-1	$\left(\frac{1}{3}\right)^{-1}=\left(\frac{3}{1}\right)^1=3$
0	1
1	$\frac{1}{3}$
2	$\left(\frac{1}{3}\right)^2=\frac{1}{9}$

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What do you notice when the base is greater than 1?

- graph always increases
- common point at (0,1)
- HA at $y=0$

exponential growth

What do you notice when the base is between 0 and 1?

- always decreasing
- point at (0,1)
- HA at $y=0$

exponential decay

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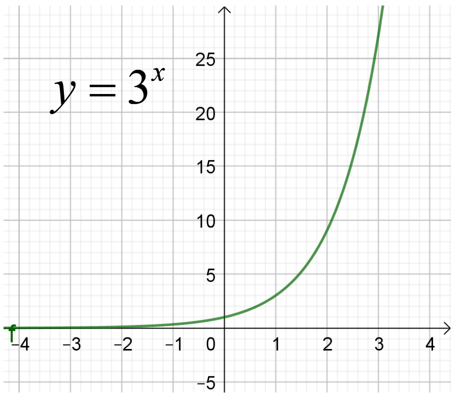
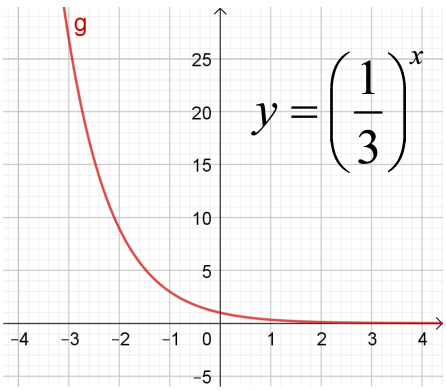
Part C - Compare graphs:

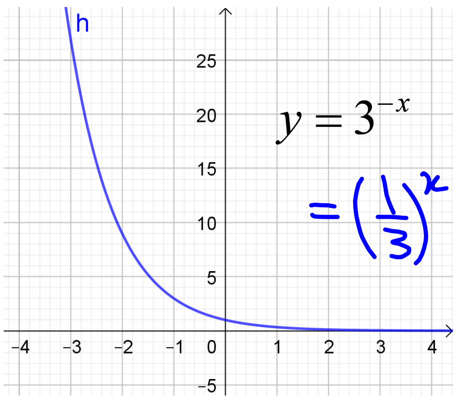
$$y=3^x$$

$$y=\left(\frac{1}{3}\right)^x$$

$$y=3^{-x}$$







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What conclusion can you draw about negative exponents?

① $3^{-x} = \left(\frac{1}{3}\right)^{x}$

② 3^{-x} is h. reflection of 3^{x}

What ordered pair (point) do they have in common? Why?

$P(0,1)$, because $a^{0} = 1$

anything, $a \neq 0$

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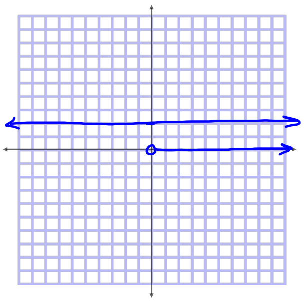
Graph 7. $y = 1^{x}$

8. $y = 0^{x}$

9. $y = (-2)^{x}$



x	$y = 1^{x}$
-2	$1^{-2} = \left(\frac{1}{1}\right)^{2} = 1$
-1	$1^{-1} = 1$
0	1
1	1
2	1

x	$y = 0^{x}$
-2	$0^{-2} = \left(\frac{1}{0}\right)^{2}$ undef
-1	0^{-1} undef
0	undef
1	$0^{1} = 0$
2	0

x	$y = (-2)^{x}$
-2	$(-2)^{-2} = \left(\frac{1}{-2}\right)^{2} = \frac{1}{4}$
-1	$(-2)^{-1} = \left(\frac{1}{-2}\right)^{1} = -\frac{1}{2}$
0	1
$\sqrt{\ } \rightarrow \frac{1}{2}$	$\sqrt{-2}$ undef
1	-2
2	4

illustrates why for $y = a^{x}$,

$0 < a < 1$ or $a > 1$

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For $y = b^x$ to be an exponential function,
b must be a positive value.
$0 < a < 1$, or $a > 1$

For $y = b^x$ to represent exponential growth,
b must be $b > 1$ and the function increases.
HA at $y = 0$

For $y = b^x$ to represent exponential decay,
b must be $0 < b < 1$ and the function decreases.
HA at $y = 0$

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