Determining Exponential Equations

Date: ______________

All horizontal transformations of exponential functions can be represented in an equivalent form using only vertical transformations and/or a different base value (*b*). Consider the following examples:

parent function	horizontal transformation(s)	transformed function	algebraic modification	equivalent change without horizontal component
$y=2^x$	reflection	$y=2^{-x}$	$y=2^{-x}$ $=\frac{1}{2^x}$ $=\left(\frac{1}{2}\right)^x$	change base to $\frac{1}{2}$ exponent becomes positive
$y=2^x$	shift left by 1	$y=2^{x+1}$	$y=2^{x+1}$ $=2^x \cdot 2^1$ $=2(2^x)$	vertical stretch by 2
$y=2^x$	compression by 2	$y=2^{2x}$	$y=2^{2x}$ $=(2^2)^x$ $=4^x$	change base to 4

Thus we will only concern ourselves with vertical transformations whenever possible. This means that when looking for the equation of an exponential function we will use the form $y=a(b)^x+q$.

Remember:

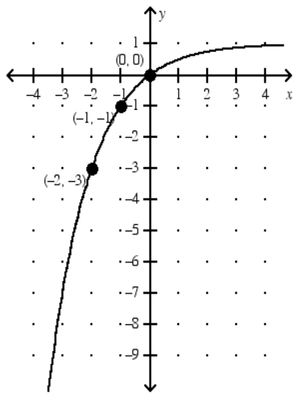
1. The *q* represents a vertical shift, which determines the *y*-value of the horizontal asymptote and affects all the other *y*-values. Always start here, if possible.
2. The *b* represents the common ratio for the exponential function. It can be determined in a few ways:
   (a) With no vertical shift ($q=0$), take the ratio of consecutive y-values ($y_2 \div y_1$).
   (b) It is also possible to take any vertical shift into account by determining the vertical distance from each consecutive point to the shifted horizontal asymptote ($d_{y_2} \div d_{y_1}$). This requires two consecutive points and the equation of the horizontal asymptote.

   Note: Consecutive points are x-values increasing by the same value each time (i.e., Δx must be constant), reading from left to right. In most cases, $\Delta x=1$. If not, our equation must be written as $y=a \cdot b^{\frac{x}{\Delta x}}+q$.

   (c) If the asymptote is unknown (not given, not available from graph), you must create a system of equations and solve for any values (a, b, or q) that are unknown.
3. Both *q* (vertical shift) and *a* (vertical scaling) affect the y-intercept, as well as other *y*-values. Once you know *q* from the horizontal asymptote, and *b* from the ratios, sub the y-intercept to find *a*.

Example 1: Determine the equation of the exponential function with a common ratio of 3, a y-intercept of 5 and a horizontal asymptote $y=-2$.

Example 2: Predict the values in $y=a(b)^x+q$, or at least their signs, from the graph. Then use the points to determine the exact equation of the exponential function and compare to your prediction.



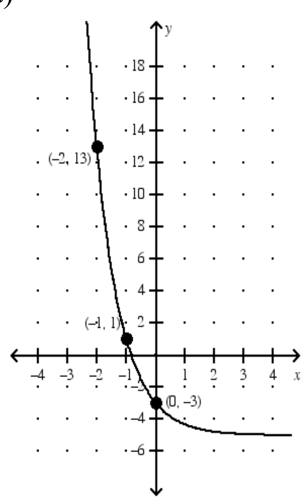
Exercises:

1. Determine the equation of the exponential function with a common ratio of 2, a horizontal asymptote $y=4$ and passing through the point (2, 10).

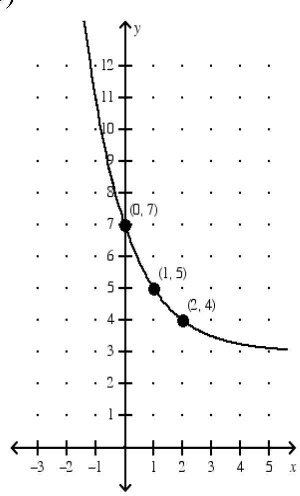
2. Determine the equation of the exponential function with a common ratio of 3, a horizontal asymptote $y=-4$ and a y-intercept of -6.

3. Determine the equations of the exponential functions represented by the graphs below:

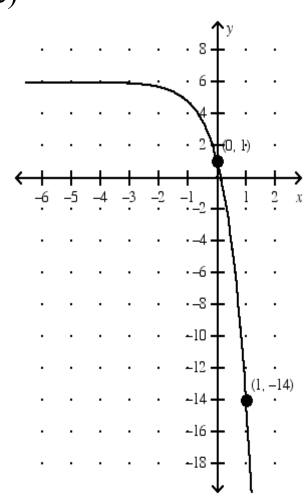
a)



b)



c)



4. Create a transformed exponential equation (vertical transformations only) and graph it. Share this graph with a partner/group and have them determine the equation