

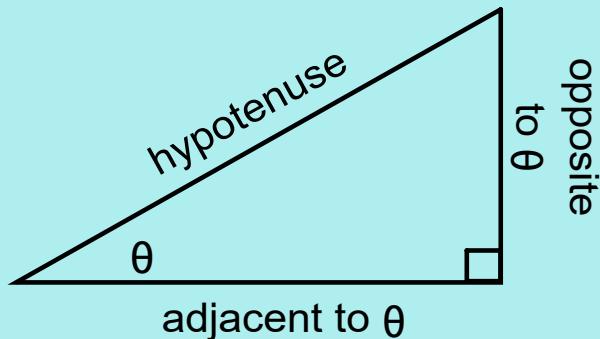
Recall:

For any angle of interest (θ), there are three (3) primary trigonometric ratios.

$$\text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

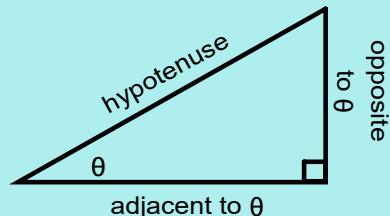
$$\text{tangent of } \theta = \frac{\text{opposite}}{\text{adjacent}}$$



S o h C a h T o a

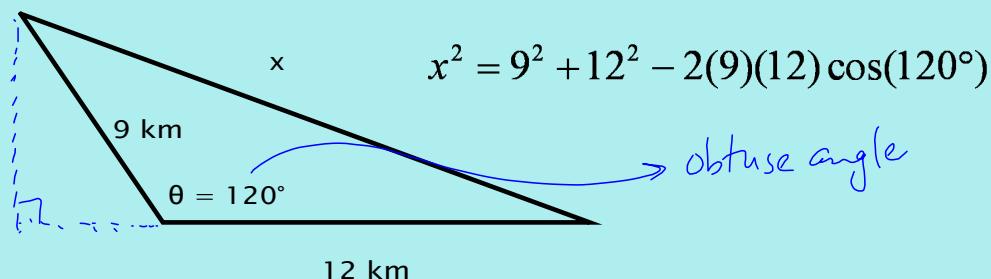
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Our reference triangle (as shown) is generally represented as an acute triangle (i.e., all angles $\leq 90^\circ$).



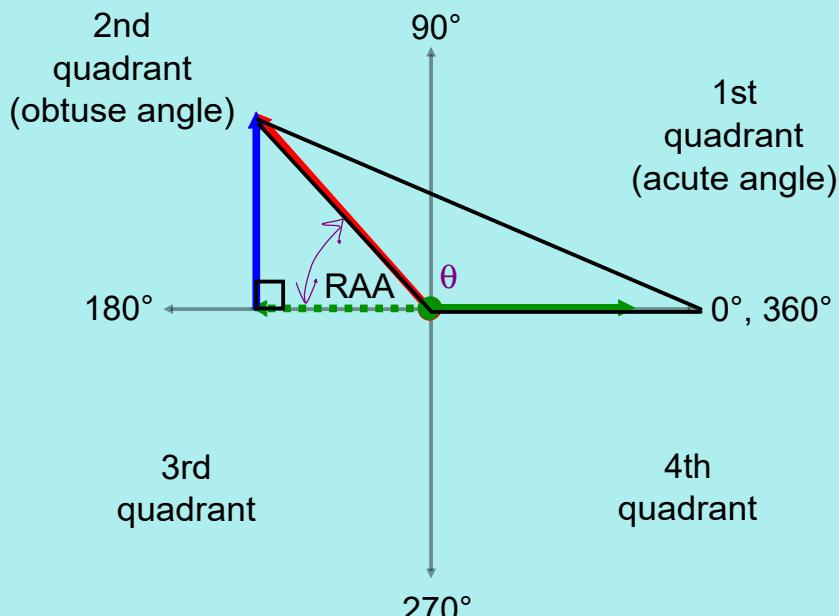
But...

using the cosine law, we have solved triangles such as the one shown below... how?



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To work with angles greater than 90° , we form a right-triangle using the terminal arm and the related acute angle.



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Trigonometry of Obtuse Angles

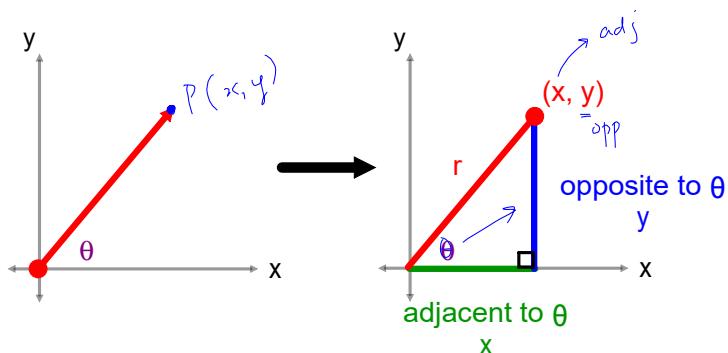
All trigonometric ratios are defined in terms of the sides of an acute right-triangle.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

Soh Cah Toa

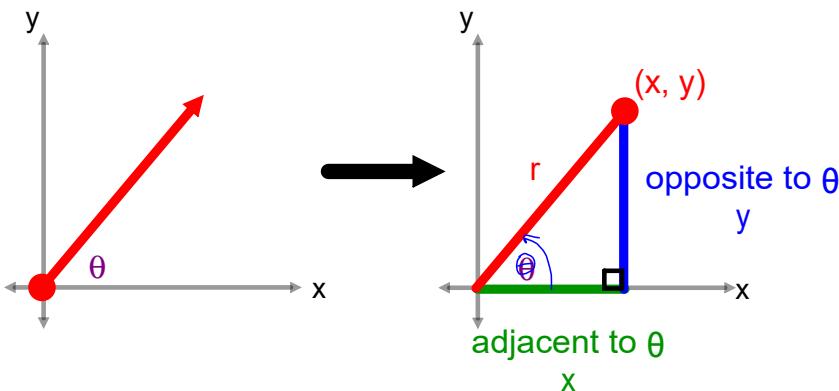
A right-angled triangle is shown with its hypotenuse labeled r . The angle at the vertex where the vertical leg meets the hypotenuse is labeled θ . The vertical leg is labeled "opposite to θ " and the horizontal leg is labeled "adjacent to θ ". A small square at the vertex indicates a right angle.

We can redefine the trig ratios for angles in standard position by drawing a right-triangle using the terminal arm.



$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$

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where: $r^2 = x^2 + y^2$, $r > 0$

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$

This definition refers only to coordinates (x, y) and the distance (r) from the origin to the point, and should be valid for any angle.

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Ex.1 The point $(-4, 3)$ is on the terminal arm of an angle θ in standard position. Find sine and cosine for θ .

$$r^2 = x^2 + y^2$$

$$r^2 = (-4)^2 + 3^2$$

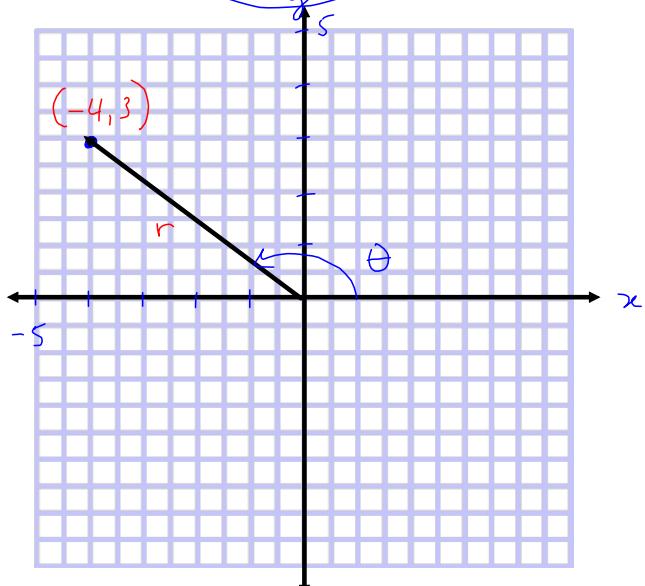
$$r^2 = 25$$

$$r = \pm \sqrt{25}, \quad r > 0$$

$$r = 5$$

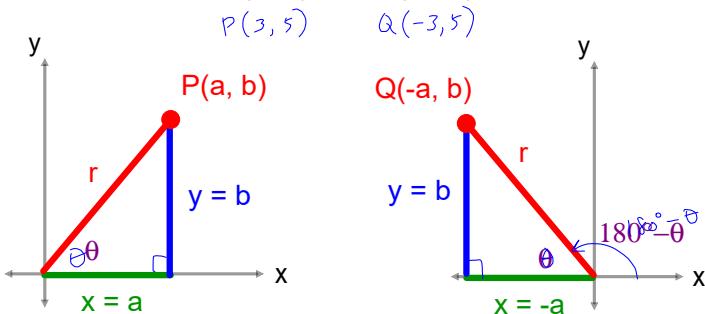
$$x = -4, \quad y = 3$$

$$\sin \theta = \frac{3}{5} \quad \cos \theta = \frac{-4}{5}$$



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Focusing on obtuse angles ($90^\circ < \theta \leq 180^\circ$), consider the points $P(a, b)$ and $Q(-a, b)$.



How can we relate $\sin \theta$ to $\sin(180^\circ - \theta)$?

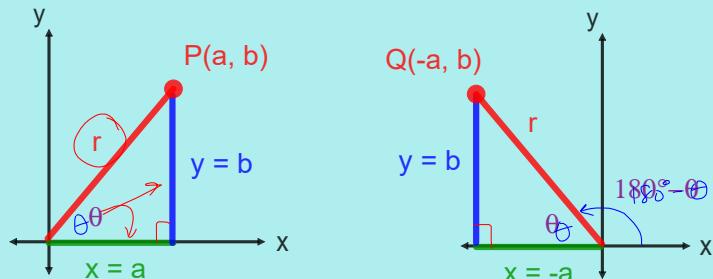
$$\begin{aligned} \sin \theta &= \frac{y}{r} & r^2 &= x^2 + y^2 \\ r &> 0 & r^2 &= (a)^2 + (b)^2 \quad \text{OR} \quad r^2 = (-a)^2 + b^2 \\ & & r^2 &= \sqrt{a^2 + b^2} & r &= \sqrt{a^2 + b^2} \end{aligned}$$

$$\sin \theta = \frac{b}{\sqrt{a^2 + b^2}} \leftrightarrow \sin(180^\circ - \theta) = \frac{b}{\sqrt{a^2 + b^2}}$$

$\therefore \sin \theta = \sin(180^\circ - \theta)$

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Focusing on obtuse angles ($90^\circ < \theta \leq 180^\circ$), consider the points $P(a, b)$ and $Q(-a, b)$.



How can we relate $\cos \theta$ to $\cos(180^\circ - \theta)$?

$$\cos \theta = \frac{x}{r} \quad r = \sqrt{a^2 + b^2}$$

$$\cos \theta = \frac{a}{\sqrt{a^2 + b^2}} \quad \cos(180^\circ - \theta) = \frac{-a}{\sqrt{a^2 + b^2}}$$

$\cos \theta = -\cos(180^\circ - \theta)$

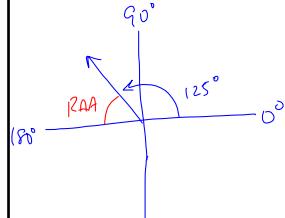
OR

$-\cos \theta = \cos(180^\circ - \theta)$

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Ex.2 Express each of the following in terms of the related acute angle, then confirm your answer.

(a) $\sin(125^\circ)$



$$\text{RAA} + 125^\circ = 180^\circ$$

$$\text{RAA} = 55^\circ$$

$$\sin \theta = \sin(180^\circ - \theta)$$

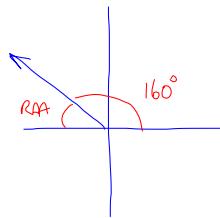
$$\sin(180^\circ - \theta) = \sin \theta$$

$$\therefore \sin 125^\circ = \sin 55^\circ$$

$$\sin 125^\circ \approx 0.8192$$

$$\sin 55^\circ \approx 0.8192$$

(b) $\cos(160^\circ)$



$$\text{RAA} = 180^\circ - 160^\circ = 20^\circ$$

$$\cos \theta = -\cos(180^\circ - \theta)$$

 \therefore

$$\cos(180^\circ - \theta) = -\cos \theta$$

$$\sqrt{\cos 160^\circ} = -\cos 20^\circ$$

$$\cos 160^\circ \approx -0.9397$$

$$-\cos 20^\circ \approx -(0.9397)$$

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Ex.3 Find θ , if $0^\circ \leq \theta \leq 180^\circ$.

(a) $\sin \theta = 0.25$

$$\theta = \sin^{-1}(0.25)$$

$$\theta \approx 14.5^\circ$$

but

$$\sin \theta = \sin(180^\circ - \theta)$$

$$\sin 14.5^\circ = \sin(180^\circ - 14.5^\circ)$$

$$\sin 14.5^\circ = \sin(165.5^\circ)$$

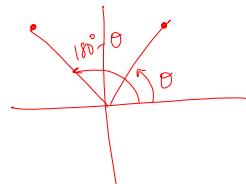
$$\therefore \theta \approx 14.5^\circ \text{ or } \theta \approx 165.5^\circ$$

\downarrow
acute obtuse

$$\sin \theta = \frac{y}{r} = 0.25$$

$$\frac{y}{r} = 0.25$$

$$r > 0, y > 0$$



(b) $\cos \theta = -0.87$

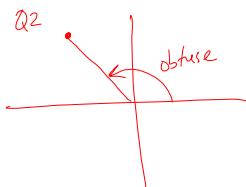
$$\theta = \cos^{-1}(-0.87)$$

$$\theta \approx 150.5^\circ \rightarrow \text{obtuse}$$

 \rightarrow no acute
answer.

$$\cos \theta = \frac{x}{r}$$

$$\frac{x}{r} = -0.87$$

but $r > 0$ $\therefore x < 0$ 

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Assigned Work:

p.281 # 1

2odd (express in terms of RAA first)

3odd, 5, 6, 9, 12*

Apr 21-12:17 AM