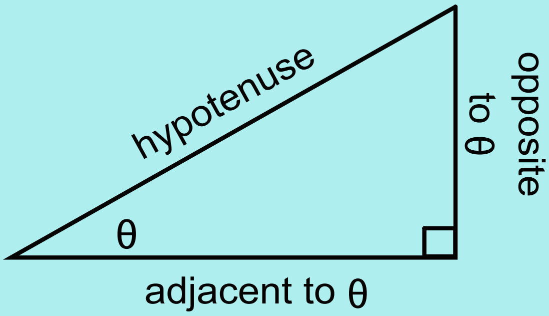
Recall:

For any angle of interest (θ), there are three (3) primary trigonometric ratios.

$$\text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

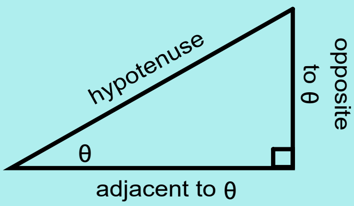
$$\text{tangent of } \theta = \frac{\text{opposite}}{\text{adjacent}}$$



S o h C a h T o a

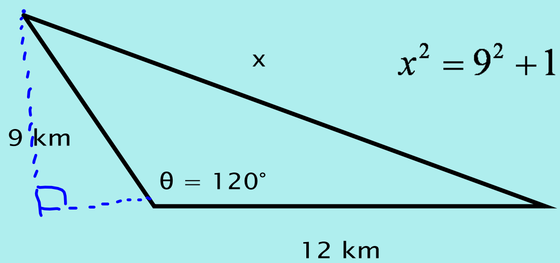
Apr 25-9:54 PM

Our reference triangle (as shown) is generally represented as an acute triangle (i.e., all angles $\leq 90°$).



But...

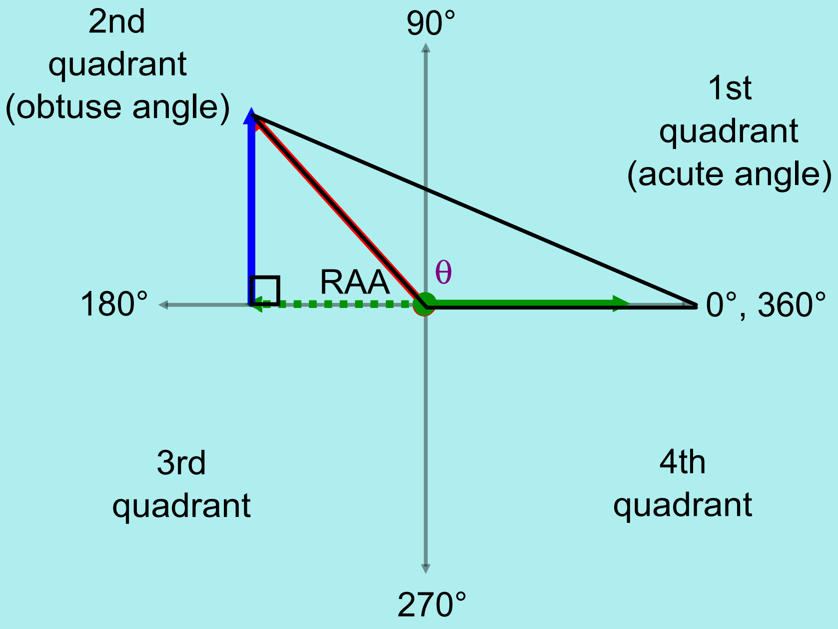
using the cosine law, we have solved triangles such as the one shown below... how?



$$x^2 = 9^2 + 12^2 - 2(9)(12)\cos(120°)$$

Apr 25-9:54 PM

To work with angles greater than 90°, we form a right-triangle using the terminal arm and the related acute angle.

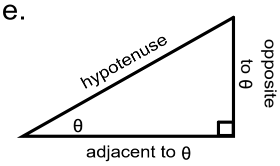


Apr 19-9:19 PM

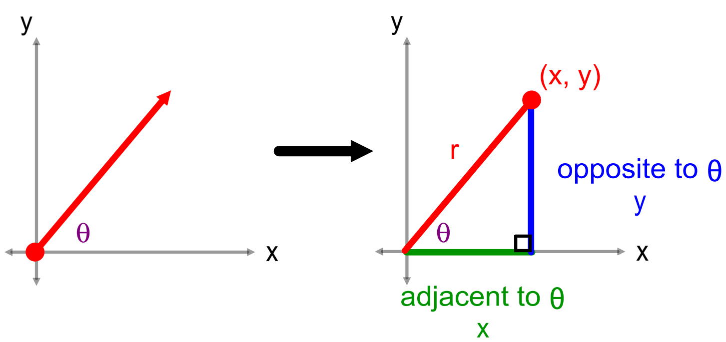
Trigonometry of Obtuse Angles

May 2/2019

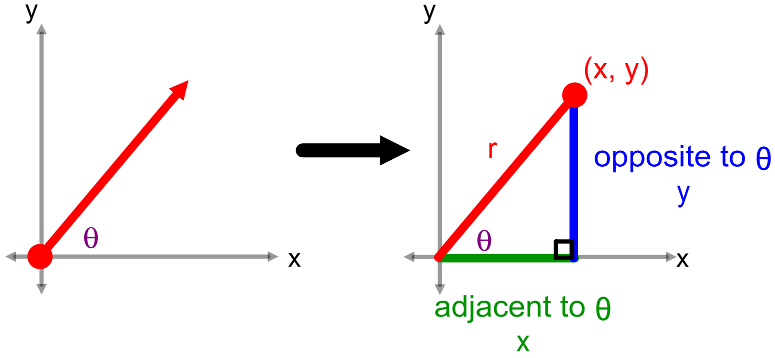
All trigonometric ratios are defined in terms of the sides of an acute right-triangle.



We can redefine the trig ratios for angles in standard position by drawing a right-triangle using the terminal arm.



Apr 19-9:13 PM



where: $r^2 = x^2 + y^2$

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

This definition refers only to coordinates (x, y) and the distance (r) from the origin to the point, and should be valid for any angle.

Apr 25-10:21 PM

Ex.1 The point (-4, 3) is on the terminal arm of an angle θ in standard position. Find sine and cosine for θ.

$x = -4$

$y = 3$

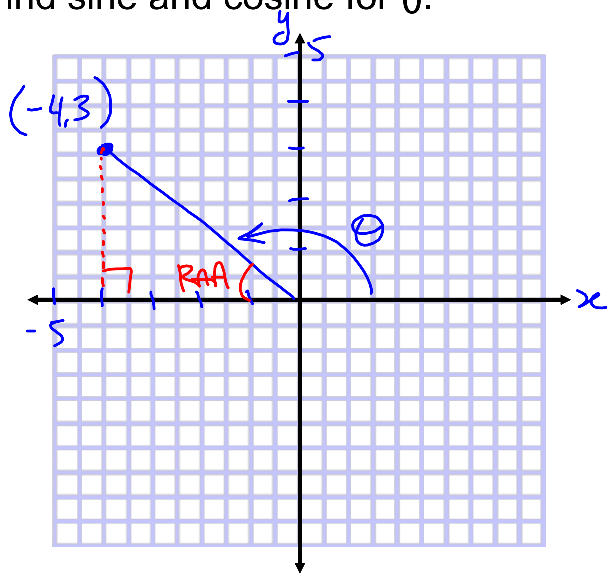
$r^2 = x^2 + y^2$

$r^2 = (-4)^2 + (3)^2$

$r^2 = 25$

$r = \pm 5,\ r > 0$

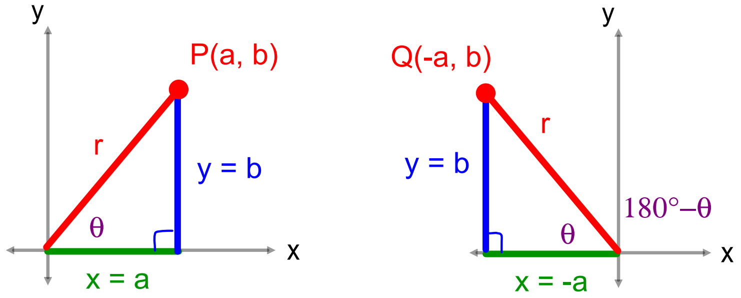
$r = 5$



$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{-4}{5}$

Apr 25-10:48 PM

Focusing on obtuse angles (90° < θ ≤ 180°), consider the points P(a, b) and Q(-a, b).



How can we relate $\sin\theta$ to $\sin(180° - \theta)$?

$r^2 = a^2 + b^2$

$r = \sqrt{a^2+b^2}, r > 0$

$\sin\theta = \frac{b}{\sqrt{a^2+b^2}}$

$r^2 = (-a)^2 + b^2$

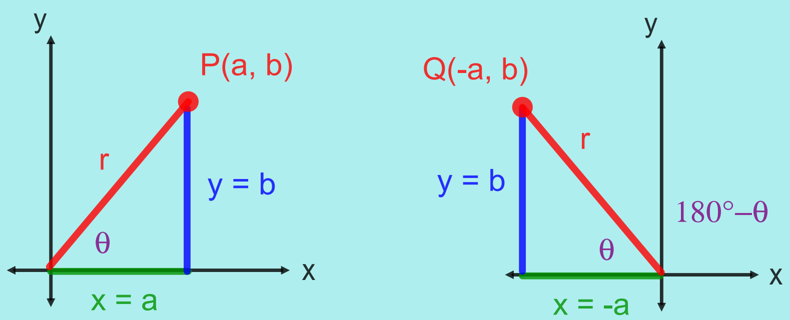
$r = \sqrt{a^2+b^2}, r > 0$

$\sin(180° - \theta) = \frac{b}{\sqrt{a^2+b^2}}$

$\therefore \sin\theta = \sin(180° - \theta)$

Apr 25-10:27 PM

Focusing on obtuse angles (90° < θ ≤ 180°), consider the points P(a, b) and Q(-a, b).



How can we relate $\cos\theta$ to $\cos(180° - \theta)$?

$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$

$\cos(180° - \theta) = \frac{-a}{\sqrt{a^2+b^2}}$

$\therefore \cos\theta = -\cos(180° - \theta)$

OR

$-\cos\theta = \cos(180° - \theta)$

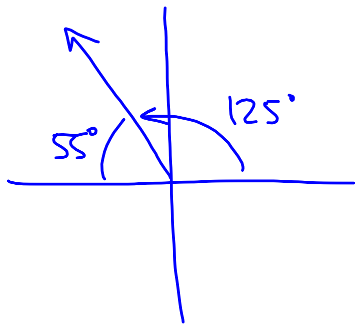
Apr 25-10:27 PM

Ex.2 Express each of the following in terms of the related acute angle, then confirm your answer.

(a) sin(125°)

$= \sin(180° - 125°)$

$= \sin(55°)$



(b) cos(160°)

$= -\cos(180° - 160°)$

$= -\cos(20°)$

Apr 25-11:14 PM

Ex.3 Find θ, if 0 ≤ θ ≤ 180°.

(a) sin θ = 0.25

$\theta = \sin^{-1}(0.25)$

$\theta \doteq 14.5°$

but

$\sin\theta = \sin(180° - \theta)$

$\sin(14.5°) = \sin(180° - 14.5°)$

$= \sin(165.5°)$

$\therefore \theta = 14.5°$ or $\theta = 165.5°$

(b) cos θ = -0.87

$\theta = \cos^{-1}(-0.87)$

$\theta \doteq 150.5°$

only answer because

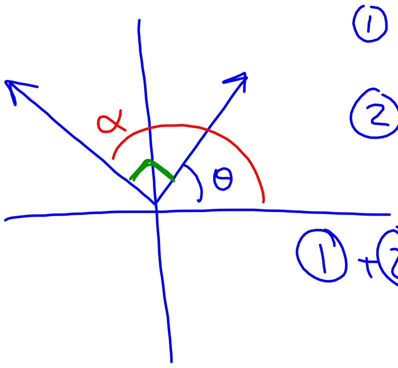
$\cos\theta \neq \cos(180° - \theta)$

Apr 25-11:14 PM

Assigned Work:

p.281 # 1
2odd (express in terms of RAA first)
3odd, 5, 6, 9, 12*

5.



① $\theta + \alpha = 180°$

② $\alpha - \theta = 90°$

①+②: $2\alpha = 270°$

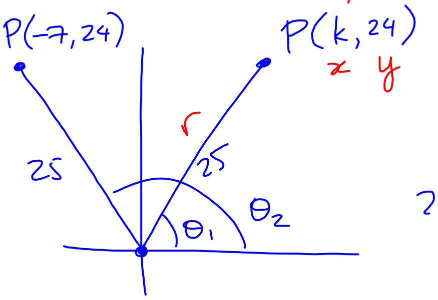
$\alpha = 135°$

$\theta + 135° = 180°$

$\theta = 45°$

Apr 21-12:17 AM

6. $P(k, 24)$



$r^2 = x^2 + y^2$

$25^2 = k^2 + 24^2$

$k^2 = 49$

$k = \pm 7$

① $k = 7$

$\sin\theta_1 = \frac{y}{r} = \frac{24}{25}$

$\cos\theta_1 = \frac{x}{r} = \frac{7}{25}$

② $k = -7$

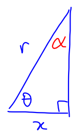
$\sin\theta_2 = \frac{y}{r} = \frac{24}{25}$

$\cos\theta_2 = \frac{x}{r} = \frac{-7}{25}$

May 3-12:35 PM

9. find θ and α so that

$\sin\theta = \cos\alpha$



$\sin\theta = \frac{y}{r} = \cos\alpha$

(a) let $\theta = 30^\circ$, $\alpha = 60^\circ$

(b) $\theta + \alpha = 90^\circ$

$\theta = 10^\circ$, $\alpha = 80^\circ$

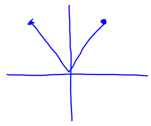
$\theta = 89^\circ$, $\alpha = 1^\circ$

(c) sine of acute > 0

sine of obtuse > 0

cosine of acute > 0

cosine of obtuse < 0



from (a) $\sin 30^\circ = \cos 60^\circ$

but $\sin 30^\circ = \sin(180^\circ - 30^\circ)$

$= \sin 150^\circ$

$\sin 150^\circ = \cos 60^\circ$

(d) $\theta - \alpha = 90^\circ$

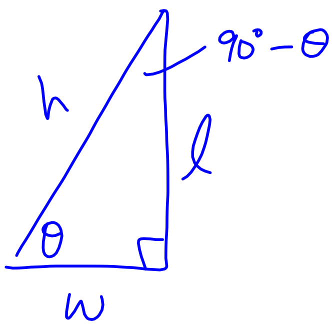
test 100° and 10°

$\sin 100^\circ$
$\doteq 0.9848$

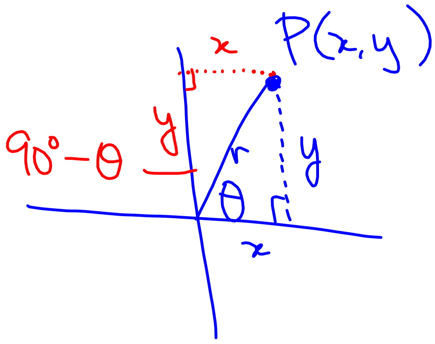
$\cos 10^\circ$
$\doteq 0.9848$

May 3-12:40 PM

12. $\sin(90^\circ - \theta) = \cos\theta$

① 

$\cos\theta = \frac{w}{h} = \sin(90^\circ - \theta)$

② 

$\cos\theta = \frac{x}{r}$

$\sin(90^\circ - \theta) = \frac{x}{r}$

May 3-12:48 PM