Recall:

For any angle θ in standard position, where the terminal arm passes through the point (x, y):

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \qquad \text{where: } r^2 = x^2 + y^2$$

For angles in Q1 and Q2,

$$\sin\theta = \sin(180° - \theta)$$

- ambiguous
- two angles give the same value for sine

$$\cos\theta = -\cos(180° - \theta)$$

or

$$-\cos\theta = \cos(180° - \theta)$$

- unambiguous
- all angles give unique values for cosine

Apr 25-9:54 PM

Sine Law & Ambiguous Case

May 3/2019

Recall: $\sin\theta = \frac{y}{r}$

y → positive in Q1, Q2

r → always positive

Q2 | Q1

Since sin θ is positive for both acute (0° to 90°) and obtuse (90° to 180°), there are two angles that yield the same answer for sin θ.

For example,

sin30° = 0.5
sin150° = 0.5

Given sin θ = 0.5, how can we choose between 30° and 150°?

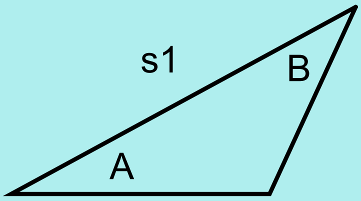
Apr 19-9:13 PM

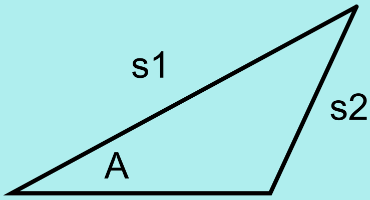
Recall: The Sine Law

The sine law is generally used when we have an oblique (non-right) triangle and:

(a) two angles and the enclosed side (ASA)

(b) two sides and a non-enclosed angle (SSA)





Apr 26-9:36 PM

When we solve for any angle using the sine law, we must consider two possible solutions, one acute and one obtuse.

Common sense will often allow us to determine which answer is appropriate. For example:

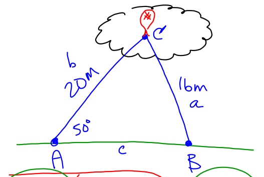
- interior angles must add to 180°
- longer sides correspond to larger angles
- geometry of situation

With SSA, it is possible to encounter three situations:
(a) no solution - a triangle cannot be formed from the data
(b) one solution - a single triangle is possible
(c) two solutions - two valid triangles can be formed

Apr 26-9:45 PM

Ex.1 Albert and Belle are launching a weather balloon into a cloud. Albert's rope is 20 m long and makes an angle of 50° with the ground. Belle's rope is 16 m.

How far is Albert from Belle? Include a sketch.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 50°}{16} = \frac{\sin B}{20}$$

$$\sin B = \frac{20 \sin 50°}{16}$$

$$B \doteq 73.2469°$$

or

$$B \doteq 180° - 73.2469°$$
$$\doteq 106.7531°$$

for $B = 73.2469°$

$$A + B + C = 180°$$
$$C = 180° - A - B$$
$$C \doteq 56.7531°$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

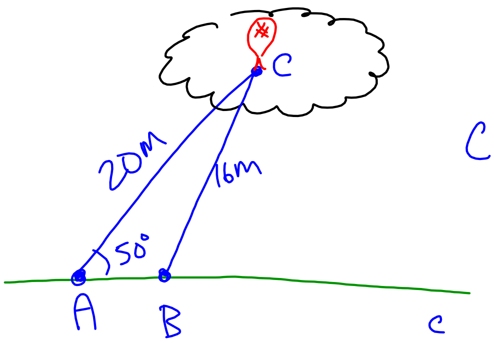
$$c = \frac{16 \sin C}{\sin 50°}$$

$$c \doteq 17.5$$

Nov 19-10:59 PM

Ex.1 Albert and Belle are launching a weather balloon into a cloud. Albert's rope is 20 m long and makes an angle of 50° with the ground. Belle's rope is 16 m.

How far is Albert from Belle? Include a sketch.



$$B \doteq 106.7531$$

$$C = 180° - A - B$$
$$\doteq 23.2469°$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{16 \sin C}{\sin 50°}$$

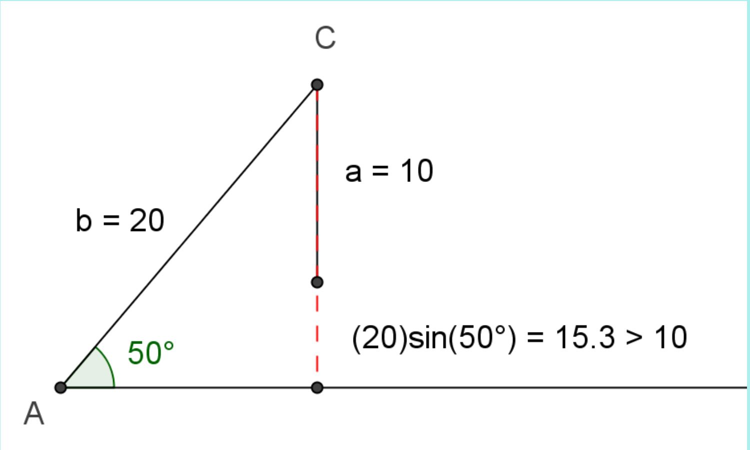
$$c \doteq 8.2$$

$\therefore$ distance between A and B is 17.5m or 8.2m.

Nov 19-10:59 PM

For acute angles (A < 90°):

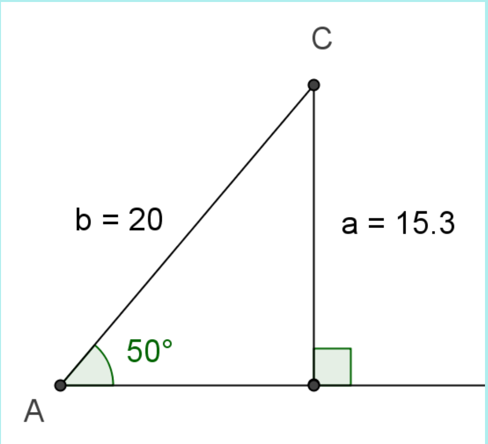
Case 1: a < b sin A
too short, no triangle



Apr 26-9:54 PM

For acute angles (A < 90°):

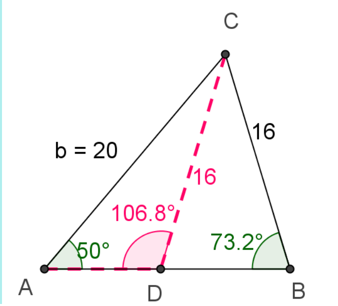
Case 2a: a = b sin A
right angle, one solution

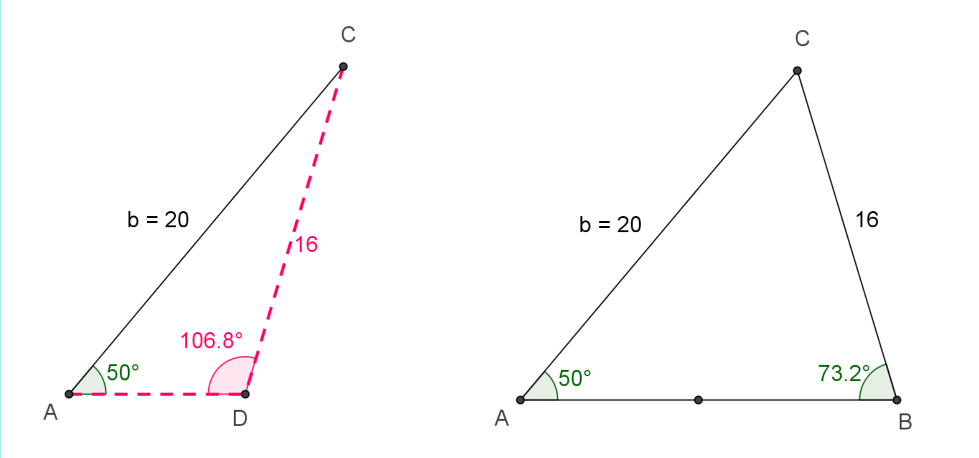


Apr 26-9:54 PM

For acute angles (A < 90°):

Case 3: a > b sin A and a < b
two valid solutions

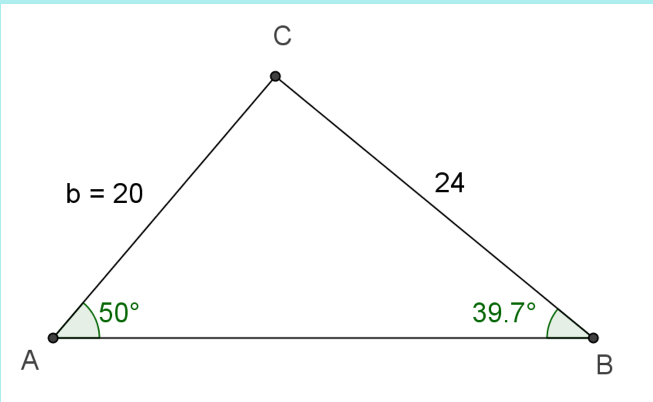




Apr 26-9:54 PM

For acute angles (A < 90°):

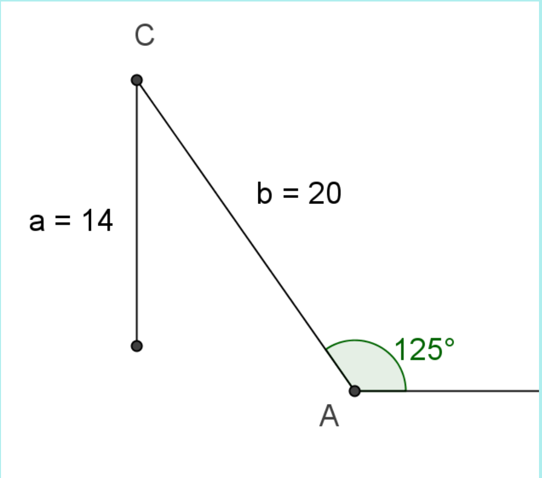
Case 2b: a $\geq$ b, one solution



Apr 26-9:54 PM

For obtuse angles ($A > 90°$):

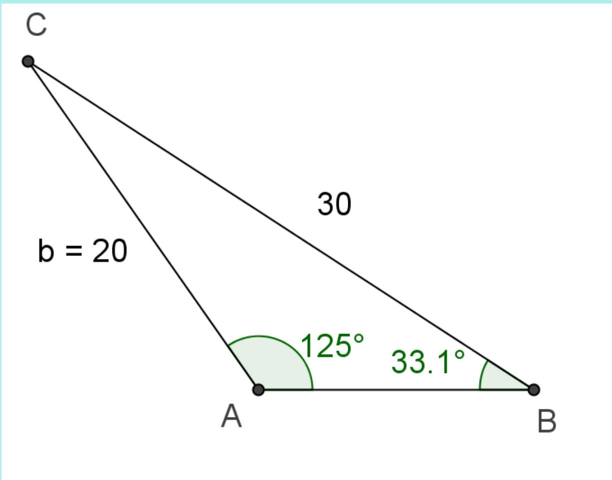
Case 1: $a \leq b$, too short, no triangle



Apr 26-9:54 PM

For obtuse angles ($A > 90°$):

Case 2: $a > b$, one solution



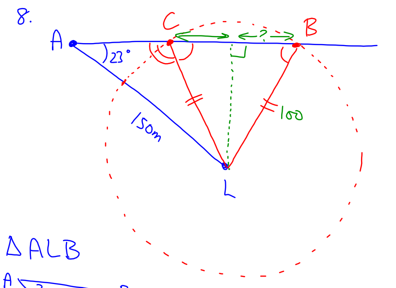
Apr 26-9:54 PM

Assigned Work:

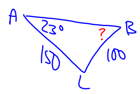
Review Ex.1 & 2 from p.303

p.308 # 1ace, 2ae, 3cg, 4, 8, 12, 13

8.



① ΔALB

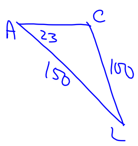


$$\frac{\sin B}{150} = \frac{\sin 23^\circ}{100}$$

$\vdots$

$B =$ ____$^\circ$ → ALB → AB

② ΔALC



C is obtuse angle from sine law

$C = 180^\circ - B$

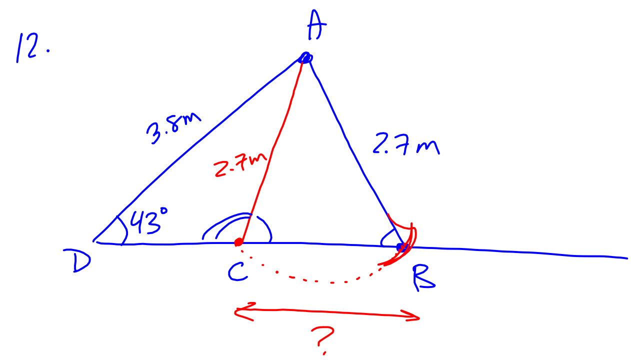
$C =$ ____

→ L → AC

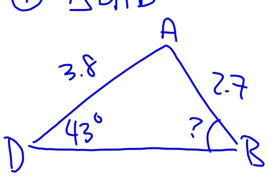
③ $CB = AB - AC$

Apr 21-12:17 AM

12.



① ΔDAB



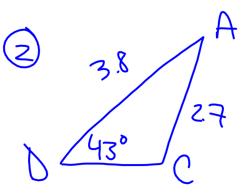
$$\frac{\sin B}{3.8} = \frac{\sin 43^\circ}{2.7}$$

$\vdots$

$B =$ ____

$\Rightarrow \angle DAB \Rightarrow \overline{DB}$

②



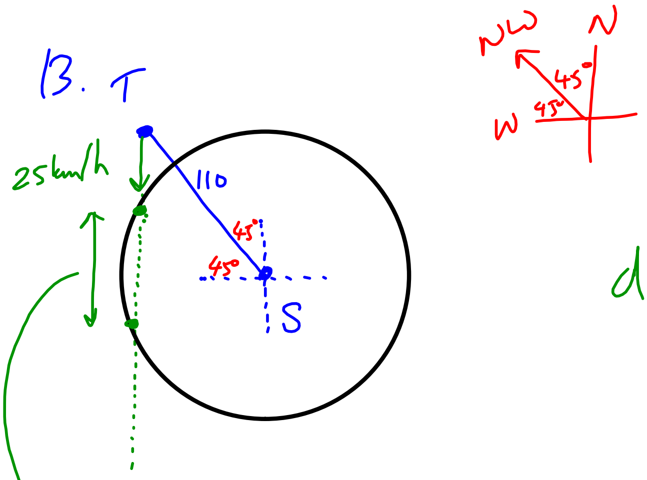
$C = 180^\circ - B$

$\Rightarrow \angle DAC \Rightarrow \overline{DC}$

③ $\overline{CB} = \overline{DB} - \overline{DC}$

May 6-2:04 PM

13\.



$d = vt \Rightarrow t = \frac{d}{v}$

distance where T is visible

$t = \frac{d}{25}$

May 6-2:10 PM