Recall:

For any angle θ in standard position, where the terminal arm passes through the point (x, y):

$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$ $\quad$ where: $r^2 = x^2 + y^2$

For angles in Q1 and Q2,

$\sin\theta = \sin(180° - \theta)$

- ambiguous
- two angles give the same value for sine

$\cos\theta = -\cos(180° - \theta)$

or

$-\cos\theta = \cos(180° - \theta)$

- unambiguous
- all angles give unique values for cosine

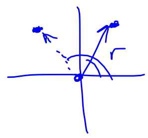
Apr 25-9:54 PM

Sine Law & Ambiguous Case

Recall: $\sin\theta = \frac{y}{r}$

y → positive in Q1, Q2

r → always positive



Since sinθ is positive for both acute (0° to 90°) and obtuse (90° to 180°), there are two angles that yield the same answer for sinθ.

For example,

sin30° = 0.5
sin150° = 0.5

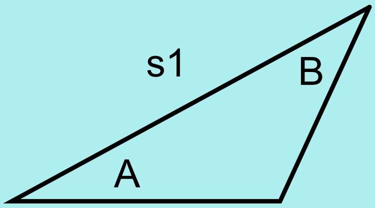
Given sinθ = 0.5, how can we choose between 30° and 150°?

Apr 19-9:13 PM

Recall: The Sine Law

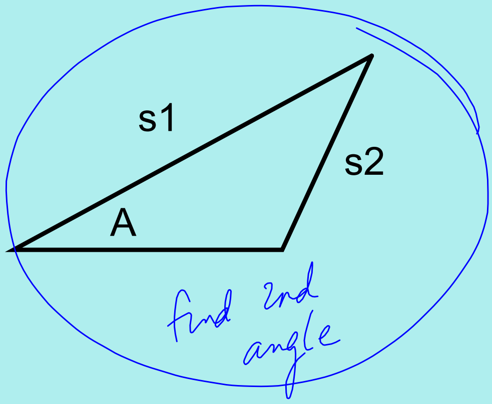
The sine law is generally used when we have an oblique (non-right) triangle and:

(a) two angles and the enclosed side (ASA)



find 2nd side

(b) two sides and a non-enclosed angle (SSA)



Apr 26-9:36 PM

When we solve for <u>any angle</u> using the sine law, we must consider two possible solutions, one acute and one obtuse.

Common sense will often allow us to determine which answer is appropriate. For example:

- interior angles must add to 180°
- longer sides correspond to larger angles
- geometry of situation

With SSA, it is possible to encounter three situations:

(a) no solution - a triangle cannot be formed from the data

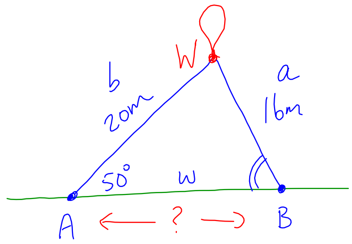
(b) one solution - a single triangle is possible

(c) two solutions - two <u>valid</u> triangles can be formed

Apr 26-9:45 PM

Ex.1 Albert and Belle are launching a weather balloon into a cloud. Albert's rope is 20 m long and makes an angle of 50° with the ground. Belle's rope is 16 m.

How far is Albert from Belle? Include a sketch.



SSA → Sine law

$$\frac{w?}{\sin W} = \frac{16}{\sin 50°} = \frac{20}{\sin B}$$

$$\frac{\sin B}{20} = \frac{\sin 50°}{16}$$

$$\sin B = \frac{20 \sin 50°}{16}$$

$B \doteq 73.2469°$

or

$B \doteq 180° - 73.2469°$

use acute value of B:

$W = 180° - 50° - B$

$W = 56.7531°$

$$\frac{w}{\sin W} = \frac{16}{\sin 50°} \quad *$$

$$w = \frac{16 \sin W}{\sin 50°}$$

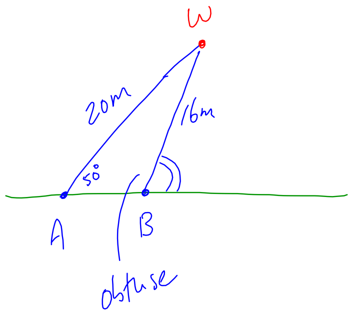
$w \doteq 17.5$

∴ Al + Belle are 17.5m apart

Nov 19-10:59 PM

Ex.1 Albert and Belle are launching a weather balloon into a cloud. Albert's rope is 20 m long and makes an angle of 50° with the ground. Belle's rope is 16 m.

How far is Albert from Belle? Include a sketch.



$$\sin B = \frac{20 \sin 50°}{16}$$

$B \doteq 106.7531°$

$W = 180° - 50° - B$

$W \doteq 23.2469°$

$$\frac{w}{\sin W} = \frac{16}{\sin 50°}$$

$$w = \frac{16 \sin W}{\sin 50°}$$

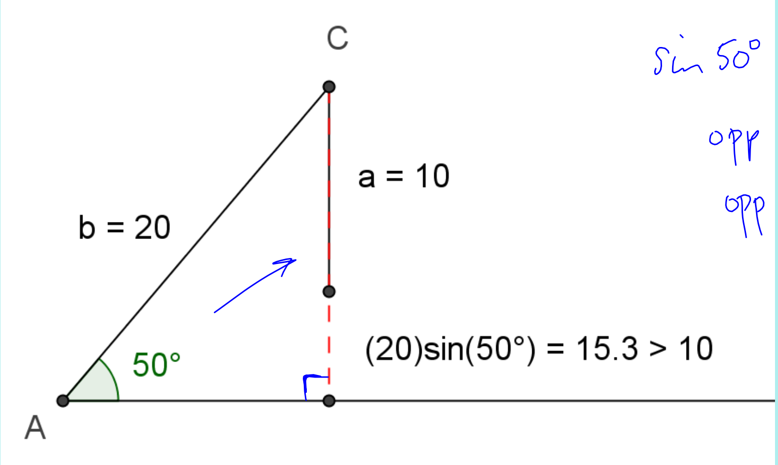
$w \doteq 8.2$

∴ Al + Belle are 8.2m apart.

Nov 19-10:59 PM

For acute angles (A < 90°):

Case 1: $a < b \sin A$
too short, no triangle, no solution



$\sin 50° = \frac{opp}{20}$

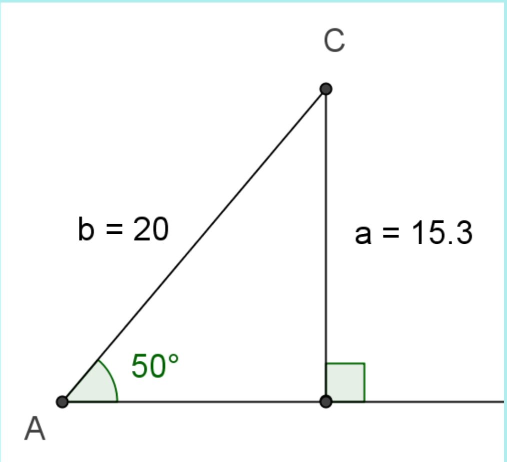
$opp = 20 \sin 50°$

$opp \doteq 15.3$

Apr 26-9:54 PM

For acute angles (A < 90°):

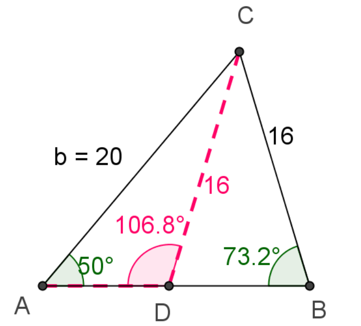
*Case 2a: $a = b \sin A$
right angle, one solution

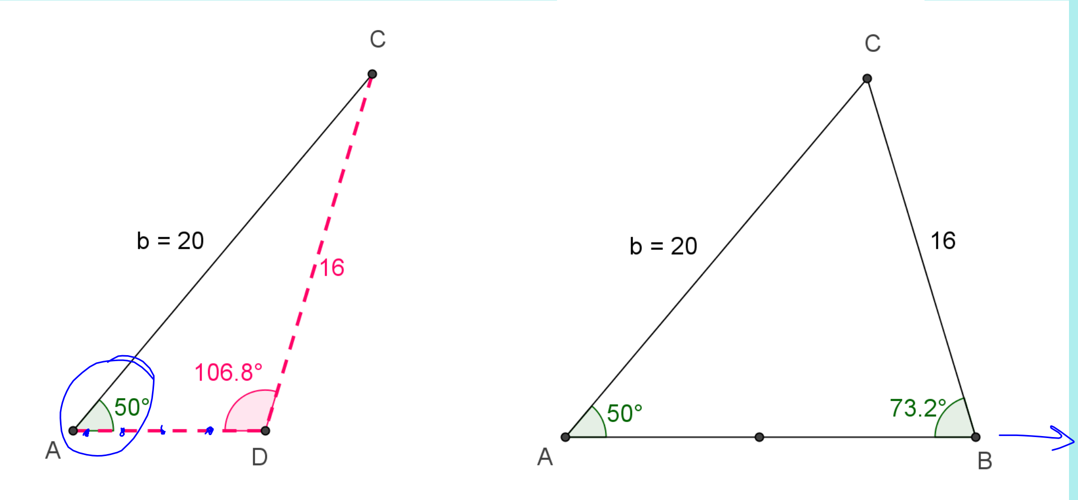


Apr 26-9:54 PM

For acute angles (A < 90°):

Case 3: a > b sin A and a < b
two valid solutions





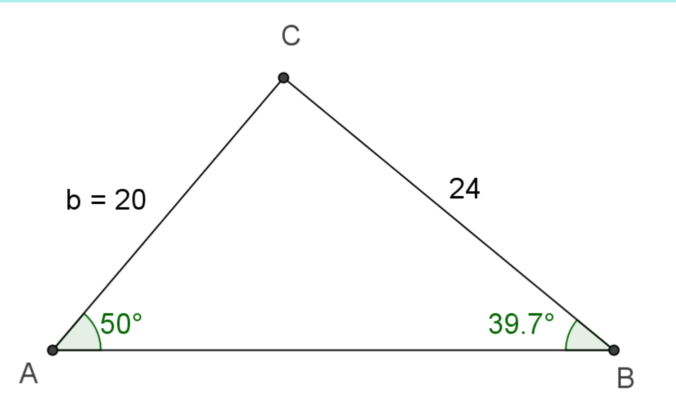
Apr 26-9:54 PM

For acute angles (A < 90°):

Case 2b: a $\geq$ b, one solution

SSA
a b ∠A

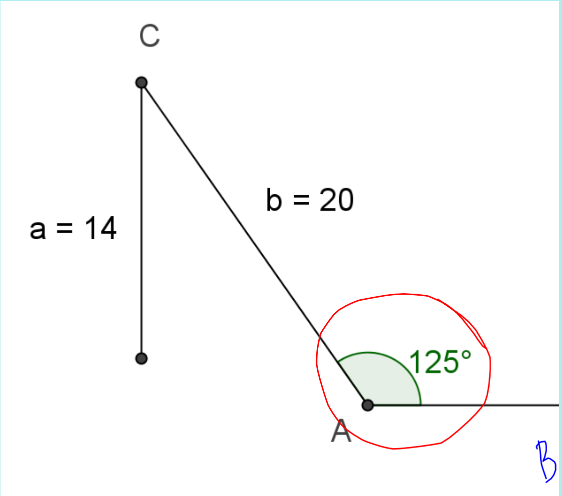
a ≥ b, one solution



Apr 26-9:54 PM

For obtuse angles (A > 90°):

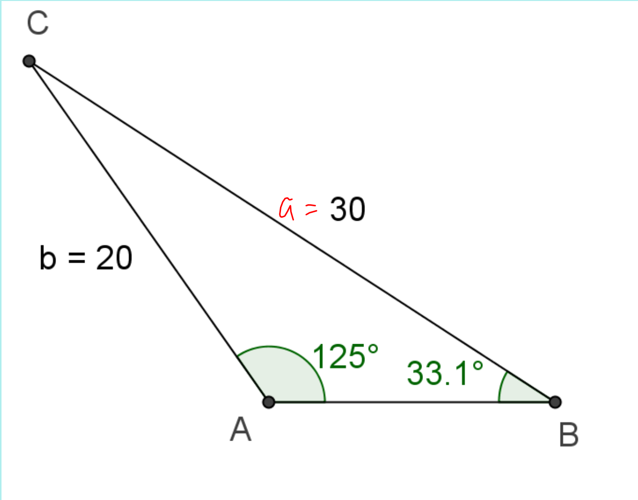
Case 1: a ≤ b, too short, no triangle



Apr 26-9:54 PM

For obtuse angles (A > 90°):

Case 2: a > b, one solution



Apr 26-9:54 PM

Assigned Work:

Review Ex.1 & 2 from p.303

p.308 # 1ace, 2ae, 3cg, 4, 8, 12, 13

Apr 21-12:17 AM