

Solving Linear Trigonometric Equations

To solve a trigonometric equation means to find the angle value(s) that satisfy the given equation.

recall: solving linear equations

Solve for x:

$$2x + 3 = 4x - 5$$

$$-2x + 5 \quad -2x + 5$$

$$8 = 2x$$

$$x = 4$$

Apr 19-9:13 PM

Solving Linear Trigonometric Equations *May 7/2019*

To solve a trigonometric equation means to find the angle value(s) that satisfy the given equation.

Steps:

- the equation should involve only one trigonometric ratio
- isolate the trigonometric ratio
- solve for the related acute angle (using positive ratio)
- use the actual sign of the ratio to determine the quadrant(s) for your answer [CAST]
- determine the angles, within your chosen quadrants, using the related acute angle

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Ex.1 Solve

$$a) \cos \theta = -\frac{\sqrt{2}}{2} \text{ where } 0^\circ \leq \theta < 360^\circ$$

① RAA

② CAST

③ θ_1, θ_2

$$\begin{aligned} \textcircled{1} \text{ RAA} &= \cos^{-1} \left(+\frac{\sqrt{2}}{2} \right) \\ &= 45^\circ \end{aligned}$$

$$\textcircled{2} \begin{array}{c|c} \text{S} & \text{A} \\ \hline \text{T} & \text{C} \end{array}$$

$$\begin{aligned} \textcircled{3} \text{ Q2: } \theta_1 &= 180^\circ - \text{RAA} \\ &= 135^\circ \end{aligned}$$

$$\begin{aligned} \text{Q3: } \theta_2 &= 180^\circ + \text{RAA} \\ &= 225^\circ \end{aligned}$$

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$$b) \sin \theta + \sqrt{3} = -\sin \theta \text{ where } 0^\circ \leq \theta < 720^\circ$$

$$\text{let } a = \sin \theta$$

$$a + \sqrt{3} = -a$$

$$2a = -\sqrt{3}$$

$$a = -\frac{\sqrt{3}}{2}$$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$2 \sin \theta = -\sqrt{3}$$

$$\begin{aligned} \textcircled{1} \text{ RAA} &= \sin^{-1} \left(+\frac{\sqrt{3}}{2} \right) \\ &= 60^\circ \end{aligned}$$

$$\textcircled{2} \begin{array}{c|c} \text{S} & \text{A} \\ \hline \text{T} & \text{C} \end{array}$$

$$\begin{aligned} \textcircled{3} \text{ Q3: } \theta_1 &= 180^\circ + \text{RAA} \\ &= 240^\circ \end{aligned}$$

$$\begin{aligned} \text{Q4: } \theta_2 &= 360^\circ - \text{RAA} \\ &= 300^\circ \end{aligned}$$

$$\text{BUT } 0^\circ \leq \theta < 720^\circ$$

$$\begin{aligned} \theta_3 &= \theta_1 + 360^\circ \\ &= 600^\circ \end{aligned}$$

$$\begin{aligned} \theta_4 &= \theta_2 + 360^\circ \\ &= 660^\circ \end{aligned}$$

need co-terminal angles

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c) $3\sin(2\theta) + 3 = 5$ where $0^\circ \leq \theta < 360^\circ$

let $\alpha = 2\theta$

$$3\sin\alpha + 3 = 5$$

$$3\sin\alpha = 2$$

$$\sin\alpha = \frac{2}{3} \rightarrow \sin 2\theta = \frac{2}{3}$$

① RAA = $\sin^{-1}\left(\frac{2}{3}\right) \approx 41.8^\circ$ ② $\frac{S}{T} \mid \frac{A}{C}$

③ Q1: $\alpha_1 = 41.8^\circ$
 Q2: $\alpha_2 = 180^\circ - \text{RAA} \approx 138.2^\circ$

BUT $0^\circ \leq \theta < 360^\circ$
 $0^\circ \leq 2\theta < 720^\circ$
 $0^\circ \leq \alpha < 720^\circ$

$$\alpha_3 = \alpha_1 + 360^\circ \approx 401.8^\circ \quad \alpha_4 = \alpha_2 + 360^\circ \approx 498.2^\circ$$

BUT $\alpha = 2\theta \Rightarrow \theta = \frac{\alpha}{2}$

$$\theta_1 \approx 20.9^\circ \quad \theta_2 \approx 69.1^\circ \quad \theta_3 \approx 200.9^\circ \quad \theta_4 \approx 249.1^\circ$$

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Assigned Work:

p.408 # 1ace, 2adf, 5ab, 14a**g**

Note: the angle π is equivalent to the angle 180°

e.g., $\frac{3\pi}{4} = \frac{3(180^\circ)}{4} = 135^\circ$

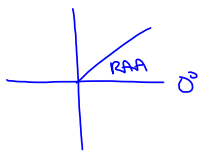
5(a) $4\cos x - 3 = 0$

$$4\cos x = 3$$

$$\cos x = \frac{3}{4}$$

① RAA = $\cos^{-1}\left(\frac{3}{4}\right) \approx 41.4^\circ$

② $\frac{S}{T} \mid \frac{A}{C}$ ③ Q1: $x \approx 41.4^\circ$
 Q4: $x = 360^\circ - \text{RAA} \approx 318.6^\circ$



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$$5(b) \quad 1 + \sin x = 4 \sin x$$

$$1 = 3 \sin x$$

$$\sin x = \frac{1}{3}$$

$$\textcircled{1} \quad \text{RAA} = \sin^{-1}\left(\frac{1}{3}\right)$$

$$= \underline{\hspace{2cm}}$$

$$\textcircled{2} \quad \begin{array}{c} \text{S} \text{ | } \text{A} \\ \hline \text{T} \text{ | } \text{C} \end{array}$$

$$\textcircled{3} \quad \text{Q1: } x = \underline{\text{RAA}}$$

$$\text{Q2: } x = 180^\circ - \text{RAA}$$

$$= \underline{\hspace{2cm}}$$

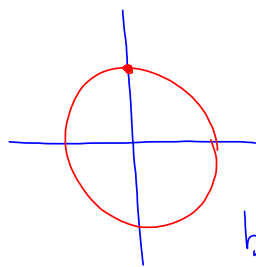
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$$14(a) \quad \sin 2x = 1$$

$$\text{let } \theta = 2x$$

$$\sin \theta = 1$$

$$\sin \theta = \frac{y}{r}$$



$$\theta = 90^\circ$$

$$2x = 90^\circ$$

$$\boxed{x_1 = 45^\circ}$$

$$\text{but } 0^\circ \leq x \leq 360^\circ$$

$$0^\circ \leq 2x \leq 720^\circ$$

$$0^\circ \leq \theta \leq 720^\circ$$

$$\theta_1 = 90^\circ$$

$$\theta_2 = 90^\circ + 360^\circ$$

$$= 450^\circ$$

$$\boxed{x_2 = \frac{450^\circ}{2}}$$

$$= 225^\circ$$

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$$14(g) \quad 2 \cos 2x = -\sqrt{3}$$

$$\cos 2x = \frac{-\sqrt{3}}{2}$$

$$\text{let } \theta = 2x$$

$$\cos \theta = \frac{-\sqrt{3}}{2}$$

$$\textcircled{1} \text{ RAA} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= 30^\circ$$

$$\textcircled{2} \begin{array}{c} \textcircled{3} A \\ \hline \textcircled{1} C \end{array} \quad \textcircled{3} \text{ Q2: } \theta_1 = 180^\circ - \text{RAA}$$

$$= 150^\circ$$

$$\text{Q3: } \theta_2 = 180^\circ + \text{RAA}$$

$$= 210^\circ$$

$$0^\circ \leq x \leq 360^\circ$$

$$0^\circ \leq 2x \leq 720^\circ$$

$$0^\circ \leq \theta \leq 720^\circ$$

$$\theta = 2x \Rightarrow x = \frac{\theta}{2}$$

$$\theta_3 = 150^\circ + 360^\circ$$

$$= 510^\circ$$

$$\theta_4 = 210^\circ + 360^\circ$$

$$= 570^\circ$$

$$x_1 = 75^\circ \quad x_2 = 105^\circ \quad x_3 = 255^\circ \quad x_4 = 285^\circ$$

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