Special Triangles & Exact Values for Trig Ratios

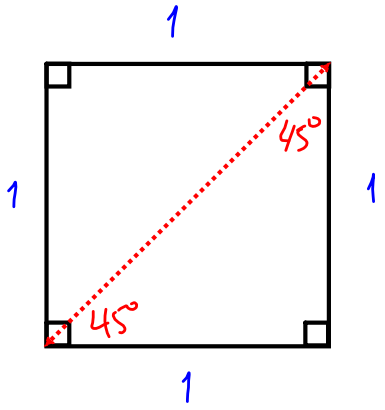
May 8/2019

Some angles occur so frequently that their trig ratios should be known exactly (i.e., no decimals, no rounding).

The angles are 30°, 45°, 60°, and 90° (and their multiples).

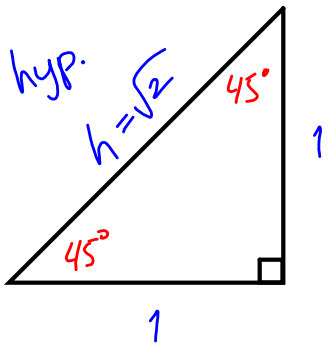
Apr 19-9:13 PM

Consider a square of side length 1.



Apr 28-11:19 PM

Now consider only ½ of the square (a right triangle)



$h^2 = 1^2 + 1^2$

$h^2 = 2$

$h = \pm\sqrt{2}$, $h > 0$

$h = \sqrt{2}$

$\sin 45° = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$

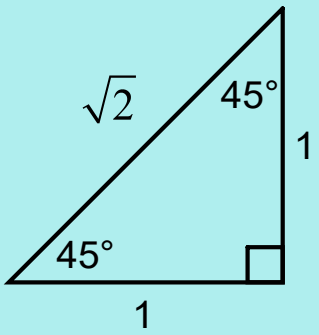
$\tan 45° = \frac{1}{1}$

$= 1$

May 1-7:53 PM

Summary:

2.The 45°-45°-90° Triangle



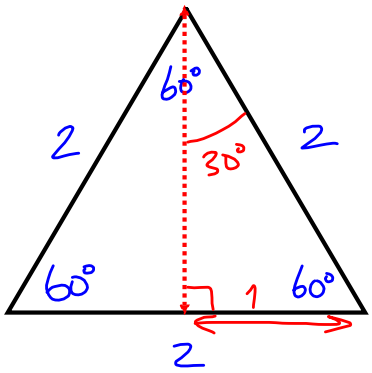
$$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = 1$$

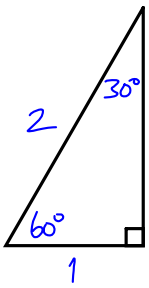
May 1-7:56 PM

Consider an equilateral triangle with a side length of 2.



May 1-7:53 PM

Now consider only ½ of the original triangle.



$y = \sqrt{3}$

$2^2 = y^2 + 1^2$

$4 = y^2 + 1$

$3 = y^2$

$y = \pm\sqrt{3}, \quad y > 0$

$y = \sqrt{3}$

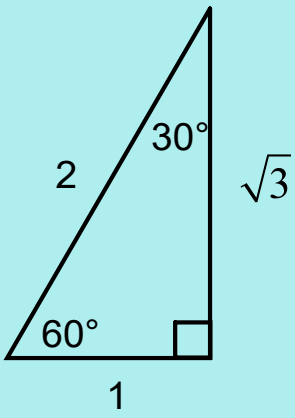
$\sin 60° = \frac{\sqrt{3}}{2} \quad \cos 60° = \frac{1}{2} \quad \tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\sin 30° = \frac{1}{2} \quad \cos 30° = \frac{\sqrt{3}}{2} \quad \tan 30° = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

May 1-7:53 PM

Summary:

1. The 30°-60°-90° Triangle



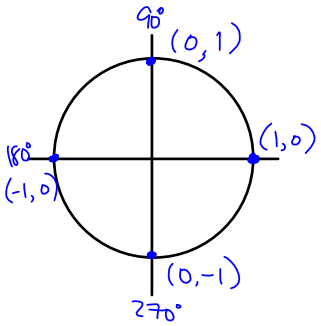
$$\sin 30° = \frac{1}{2} \qquad \sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{1}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \tan 60° = \sqrt{3}$$

May 1-7:56 PM

What about multiples of 90°?



$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

The unit circle is centred at the origin with radius, $r = 1$.

$\sin 0° = \frac{0}{1} = 0$ $\cos 0° = \frac{1}{1} = 1$ $\tan 0° = \frac{0}{1} = 0$

$\sin 90° = 1$ $\cos 90° = 0$ $\tan 90° = \frac{1}{0}$ undefined

$\sin 180° = 0$ $\cos 180° = -1$ $\tan 180° = 0$

$\sin 270° = -1$ $\cos 270° = 0$ $\tan 270°$ undef

note: $P(x, y) = P(\cos\theta, \sin\theta)$

May 3-9:34 AM

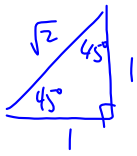
Ex.1 Evaluate using exact values and express your answer in reduced form.

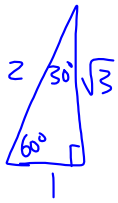
(a) $\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

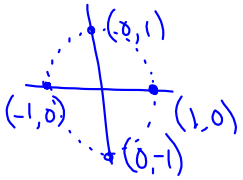
(b) $\cos^2 60° = (\cos 60°)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

(c) $\cos 45° + \sin 30° = \frac{\sqrt{2}}{2} + \frac{1}{2} = \frac{1+\sqrt{2}}{2}$

(d) $2\tan 60° - 3\tan 30° = 2\left(\sqrt{3}\right) - 3\left(\frac{1}{\sqrt{3}}\right) = 2\sqrt{3} - \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 2\sqrt{3} - \frac{3\sqrt{3}}{3} = \sqrt{3}$







Apr 28-11:19 PM

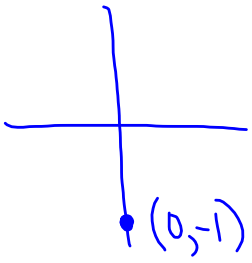
Ex.2 Evaluate

(a) $\sin 315° = -\sin 45° = \frac{-\sqrt{2}}{2}$

(b) $\tan 300°$

(c) $\cos 180°$

(d) $\tan 270°$ undefined



May 2-9:50 PM

Assigned Work:

Complete Handout Questions,
p.348 # 3

Ho 2k, 3f, 1e

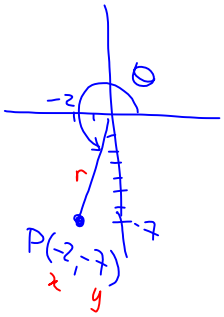
1(e) $P(-2,-7)$

$r^2 = (-2)^2 + (-7)^2$

$r^2 = 4 + 49$

$r = \pm\sqrt{53}, \ r > 0$

$r = \sqrt{53}$



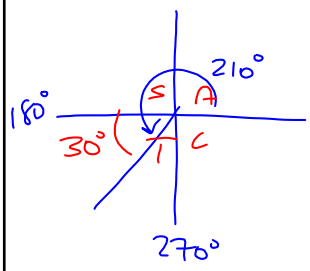
$\sin\theta = \frac{-7}{\sqrt{53}} = \frac{-7\sqrt{53}}{53}$ $\cos\theta = \frac{-2}{\sqrt{53}} = \frac{-2\sqrt{53}}{53}$ $\tan\theta = \frac{-7}{-2} = \frac{7}{2}$

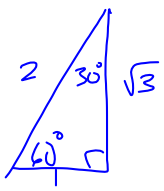
Apr 21-12:17 AM

2(b) $\cos 210^\circ = -\cos 30^\circ$

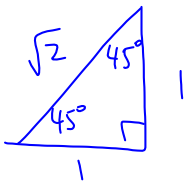
$= -\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{-\sqrt{3}}{2}$





3(e) $\cos\theta = \frac{1}{\sqrt{2}}$



① $RAA = 45^\circ$

② S | A
T | C

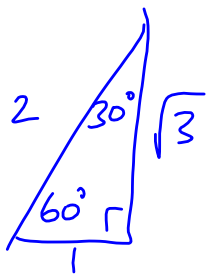
Q1: $\theta = 45^\circ$

Q4: $\theta = 360^\circ - 45^\circ$

$= 315^\circ$

May 9-12:39 PM

3 (f) $\sin\theta = \frac{-\sqrt{3}}{2}$



① $RAA = 60°$

② S | A

I C

③ Q3: $\theta = 180° + RAA$

$= 240°$

Q4: $\theta = 360° - RAA$

$= 300°$

May 9-12:43 PM