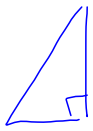
Special Triangles & Exact Values for Trig Ratios

Some angles occur so frequently that their trig ratios should be known exactly (i.e., no decimals, no rounding).

The angles are 30°, 45°, 60°, and 90° (and their multiples).

acute



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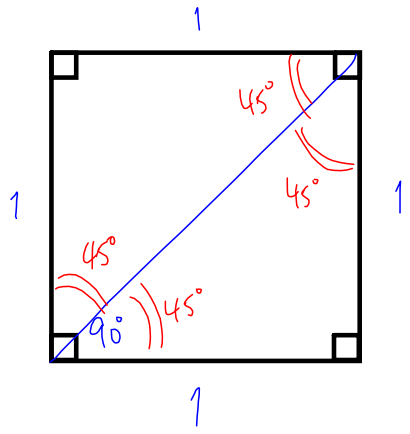
Consider a square of side length 1.



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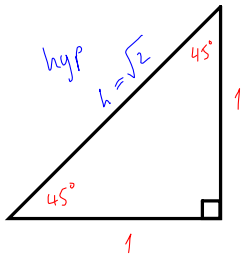
Now consider only ½ of the square (a right triangle)



$h^2 = 1^2 + 1^2$

$h^2 = 2$

$h = \pm\sqrt{2}$, $h > 0$

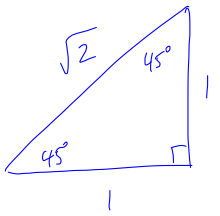
$h = \sqrt{2}$

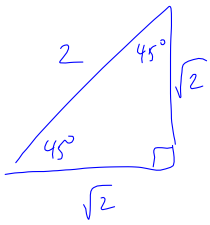
$\sin 45° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}}$

$\cos 45° = \frac{1}{\sqrt{2}}$

$\tan 45° = \frac{1}{1} = 1$

$\sin 45° = \frac{\sqrt{2}}{2}$ $\cos 45° = \frac{\sqrt{2}}{2}$ $\tan 45° = 1$





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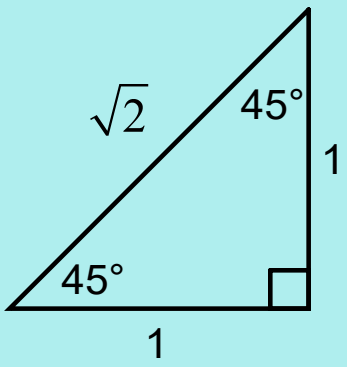
Summary:

2.The 45°-45°-90° Triangle



$$\sin 45° = \frac{1}{\sqrt{2}} \qquad \cos 45° = \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2} \leftarrow \text{final answer} \rightarrow = \frac{\sqrt{2}}{2}$$

$$\tan 45° = 1$$

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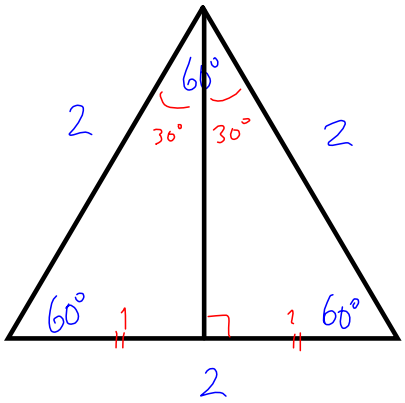
Consider an equilateral triangle with a side length of 2.



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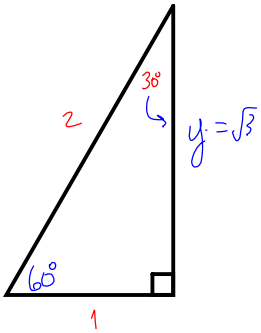
Now consider only ½ of the original triangle.



$2^2 = y^2 + 1^2$

$4 = y^2 + 1$

$y^2 = 3$

$y = \pm\sqrt{3}, \ y > 0$

$y = \sqrt{3}$

$\sin 60° = \frac{\sqrt{3}}{2}$ $\quad \cos 60° = \frac{1}{2}$ $\quad \tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\sin 30° = \frac{1}{2}$ $\quad \cos 30° = \frac{\sqrt{3}}{2}$ $\quad \tan 30° = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

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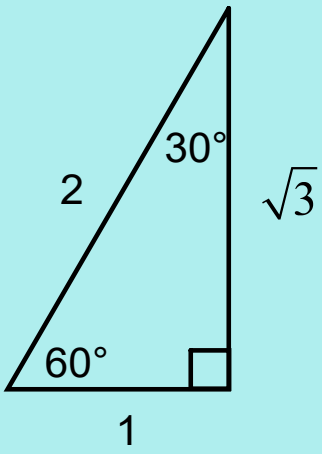
Summary:

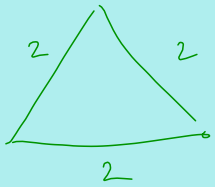
1. The 30°-60°-90° Triangle



$$\sin 30° = \frac{1}{2} \qquad \sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{1}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \tan 60° = \sqrt{3}$$



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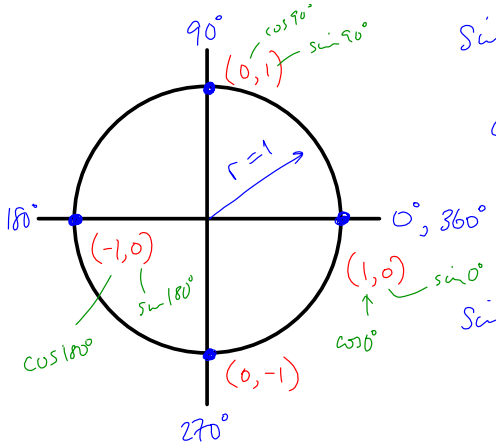
What about right angles?



$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

choose $P(x,y)$ on unit circle $(r = 1)$

$\sin 0° = \frac{0}{1} = 0$ $\cos 0° = \frac{1}{1} = 1$ $\tan 0° = \frac{0}{1} = 0$

$\sin 90° = 1$ $\sin 180° = 0$ $\sin 270° = -1$

$\cos 90° = 0$ $\cos 180° = -1$ $\cos 270° = 0$

$\tan 90°$ undefined $\tan 180° = 0$ $\tan 270°$ undef.

In general, on unit circle,

$$P(x,y) = P(\cos\theta, \sin\theta)$$

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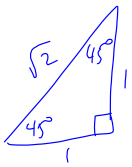
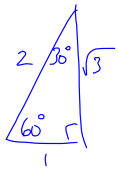
Ex.1 Evaluate using exact values and express your answer in reduced form.

(a) $\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



(b) $\cos^2 60° = (\cos 60°)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$



(c) $\cos 45° + \sin 30° = \frac{\sqrt{2}}{2} + \frac{1}{2} = \frac{\sqrt{2}+1}{2} = \frac{1+\sqrt{2}}{2}$

(d) $2\tan 60° - 3\tan 30° = 2\left(\frac{\sqrt{3}}{1}\right) - 3\left(\frac{1}{\sqrt{3}}\right) = 2\sqrt{3} - \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 2\sqrt{3} - \frac{3\sqrt{3}}{3} = 2\sqrt{3} - 1\sqrt{3} = \sqrt{3}$

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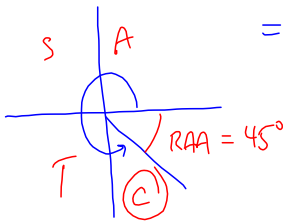
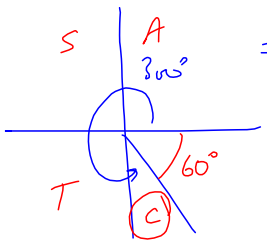
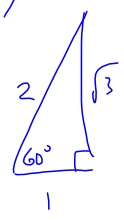
Ex.2 Evaluate

(a) $\sin 315° = -\sin 45°$ (CAST, RAA) $= -\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{2}}{2}$

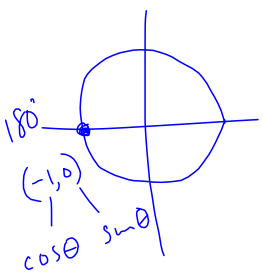


(b) $\tan 300° = -\tan 60° = -\left(\frac{\sqrt{3}}{1}\right) = -\sqrt{3}$

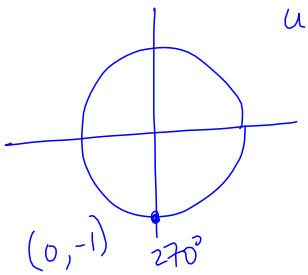




(c) $\cos 180° = \frac{x}{r} = \frac{-1}{1} = -1$



(d) $\tan 270° = \frac{y}{x}$ undefined



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Assigned Work:

Complete Handout Questions,
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