Recall:

In the x-y plane, trig ratios are expressed in terms of x, y, and r.

P(x,y)

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \qquad \text{where: } r^2 = x^2 + y^2$$

An identity is an equation which is always true for all values of the variable.

Quotient Identity: $$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Pythagorean Identity: $$\sin^2\theta + \cos^2\theta = 1$$

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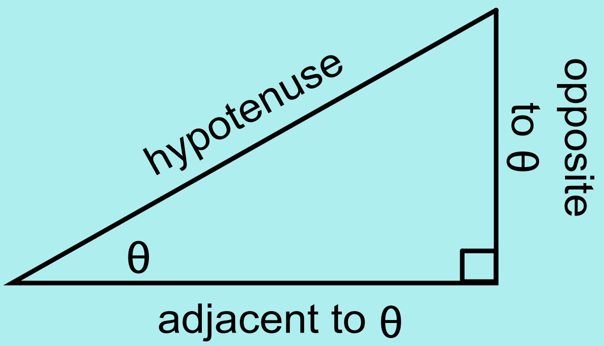
Recall:

For any angle of interest (θ), there are three (3) primary trigonometric ratios.

$$\text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent of } \theta = \frac{\text{opposite}}{\text{adjacent}}$$



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Trigonometric Identities (continued)

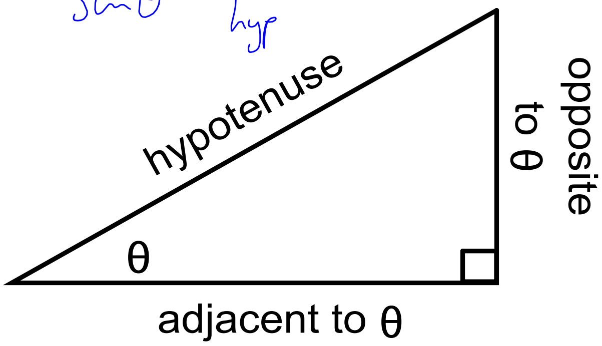
The reciprocal identities are defined as follows:

$\sin\theta = \frac{opp}{hyp}$

$$\text{cosecant of } \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\text{secant of } \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\text{cotangent of } \theta = \frac{\text{adjacent}}{\text{opposite}}$$



They are the reciprocals of the fundamental ratios:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

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Using the reciprocal identities, consider dividing $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$.

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$$\frac{1}{\cos\theta} = \sec\theta$$

$$\left(\frac{1}{\cos\theta}\right)^2 = \sec^2\theta$$

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Using the reciprocal identities, consider dividing $\sin^2\theta+\cos^2\theta=1$ by $\sin^2\theta$.

$$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$\boxed{1+\cot^2\theta=\csc^2\theta}$$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}$$

$$\frac{1}{\tan\theta}=\frac{\cos\theta}{\sin\theta}$$

$\cot\theta$

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Summary:

Quotient Identity:

$$\tan\theta=\frac{\sin\theta}{\cos\theta}$$

Pythagorean Identity:

$$\sin^2\theta+\cos^2\theta=1$$

Reciprocal Identities: by definition

$$\csc\theta=\frac{1}{\sin\theta} \qquad \sec\theta=\frac{1}{\cos\theta} \qquad \cot\theta=\frac{1}{\tan\theta}$$

Other Useful Identities:

$$\tan^2\theta+1=\sec^2\theta \qquad \cot^2\theta+1=\csc^2\theta$$

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Ex.1 Prove $\csc\theta = \dfrac{\cot\theta}{\cos\theta}$ (#1 from WS 3.3)

LS RS

$$RS = \frac{\cot\theta}{\cos\theta}$$

$$= \frac{\frac{\cos\theta}{\sin\theta}}{\left(\frac{\cos\theta}{1}\right)}$$

$$= \frac{\cancel{\cos\theta}^{\,1}}{\sin\theta} \cdot \frac{1}{\cancel{\cos\theta}_{\,1}}$$

$$= \frac{1}{\sin\theta}$$

$$= \csc\theta$$

$$= LS$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta}{\sin\theta}$$

$\therefore$ identity is true.

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Ex.2 Prove $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$
(#5 from WS 3.3)

RS

$$LS = \csc^4 x - \cot^4 x$$

$$= \underbrace{(\csc^2 x - \cot^2 x)}_{1}\underbrace{(\csc^2 x + \cot^2 x)}_{RS}$$

let $a = \csc x$
$b = \cot x$

$LS = a^4 - b^4$

let $p = a^2$
$q = b^2$

$$LS = p^2 - q^2$$
$$= (p-q)(p+q)$$
$$= (a^2-b^2)(a^2+b^2)$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$1 = \csc^2\theta - \cot^2\theta$$

$$= (1)(\csc^2 x + \cot^2 x)$$
$$= RS$$

$LS = RS$ ✓

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Assigned Work:

WS Part A # 2, 3, 4, 6, 9, 11, 12, 14, 15

Apr 21-12:17 AM