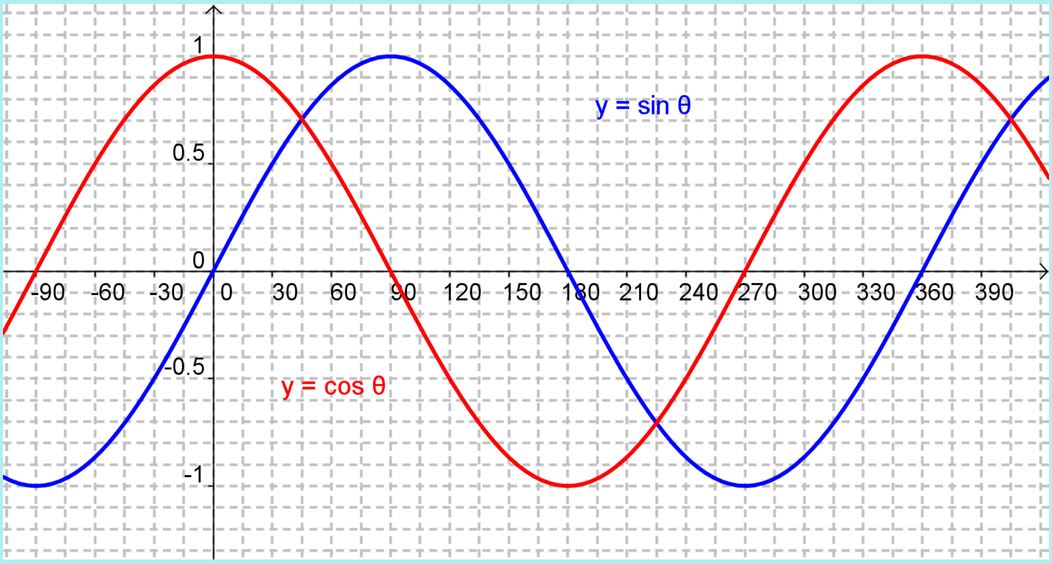
Graphing Transformed Trigonometric Functions

Recall: parent functions for sine and cosine



May 21-8:27 PM

Graphing Transformed Trigonometric Functions May 21/2019

Recall: $y = af[k(x-p)]+q$

For sinusoidal functions, this becomes

$$y = a\sin\left[k\left(x-p\right)\right]+q$$ for $f(x) = \sin(x)$

or

$$y = a\cos\left[k\left(x-p\right)\right]+q$$ for $f(x) = \cos(x)$

May 17-9:17 AM

To graph a transformed function, you can transform key points on the parent function using:

$$y = af[k(x-p)] + q$$

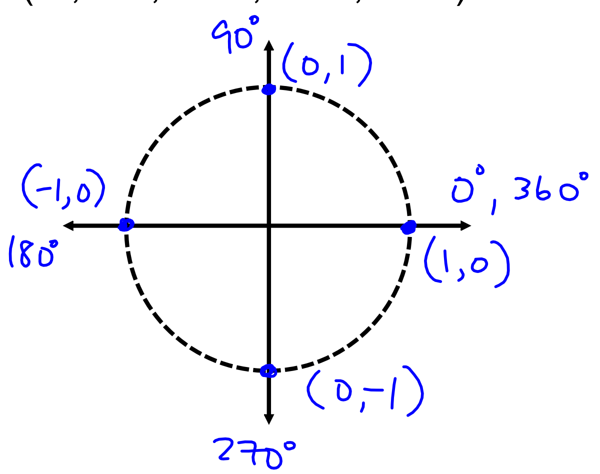
a gives vertical reflection and scaling
k gives horizontal reflection and scaling
p gives horizontal translation or shift
q gives vertical translation or shift

$$(x,y) \rightarrow (x,ay) \rightarrow \left(\frac{x}{k},ay\right) \rightarrow \left(\frac{x}{k}+p,ay\right) \rightarrow \left(\frac{x}{k}+p,ay+q\right)$$

$$y = 3\sin\left[\frac{1}{2}(\underbrace{x+5}_{x-(-5)})\right] - 2$$

May 17-9:23 AM

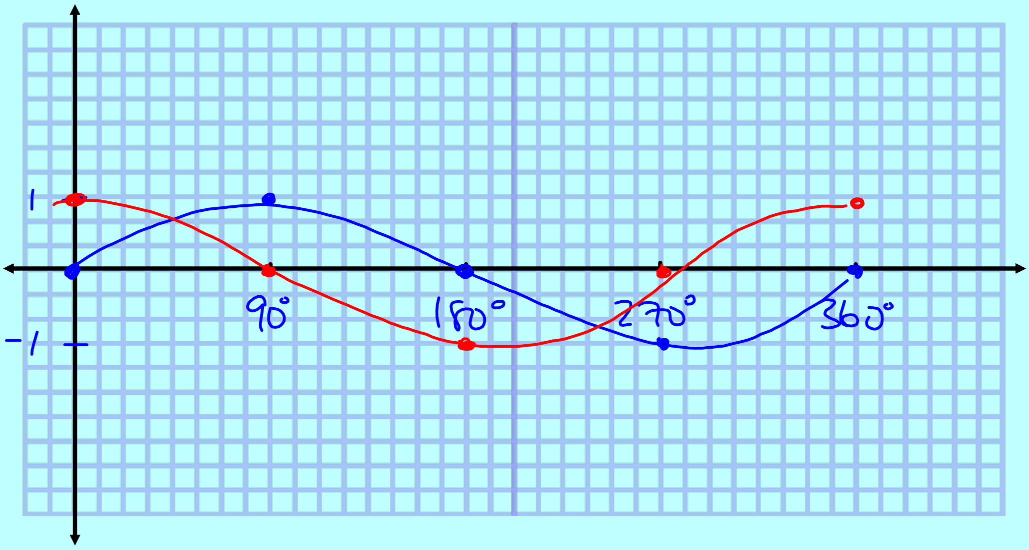
The simplest way to sketch the parent function for sine or cosine is to use 5 key points at 90° intervals (0°, 90°, 180°, 270°, 360°).



θ	$\sin\theta$	$\cos\theta$
0°	0	1
90°	1	0
180°	0	-1
270°	-1	0
360°	0	1

May 17-9:19 AM

The simplest way to sketch the parent function for sine or cosine is to use 5 key points at 90º intervals (0º, 90º, 180º, 270º, 360º).



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(b) graph using key properties

a - vertical reflection and amplitude
k - horizontal reflection and period

$$period = \frac{360°}{k}$$

p - phase shift of starting point
q - axis of the curve, $y = q$

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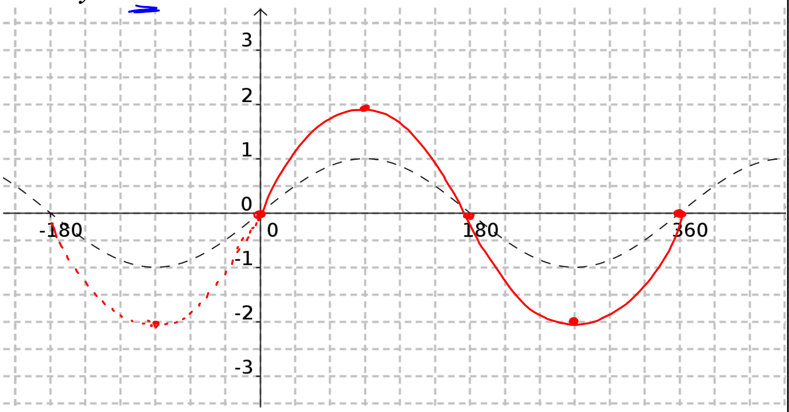
Ex.1 See handout

(a) $y = 2\sin x$

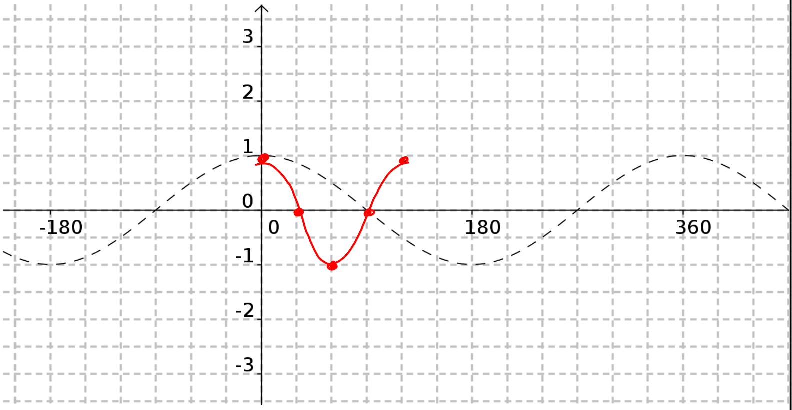


$a = 2 \longrightarrow$ V. stretch by 2 $\quad y \times 2$

$(0, 0) \longrightarrow (0, 0)$
$(90°, 1) \longrightarrow (90°, 2)$
$(180°, 0) \longrightarrow (180°, 0)$
$(270°, -1) \longrightarrow (270°, -2)$
$(360°, 0) \longrightarrow (360°, 0)$

Dec 6-10:55 AM

(b) $y = \cos(3x)$

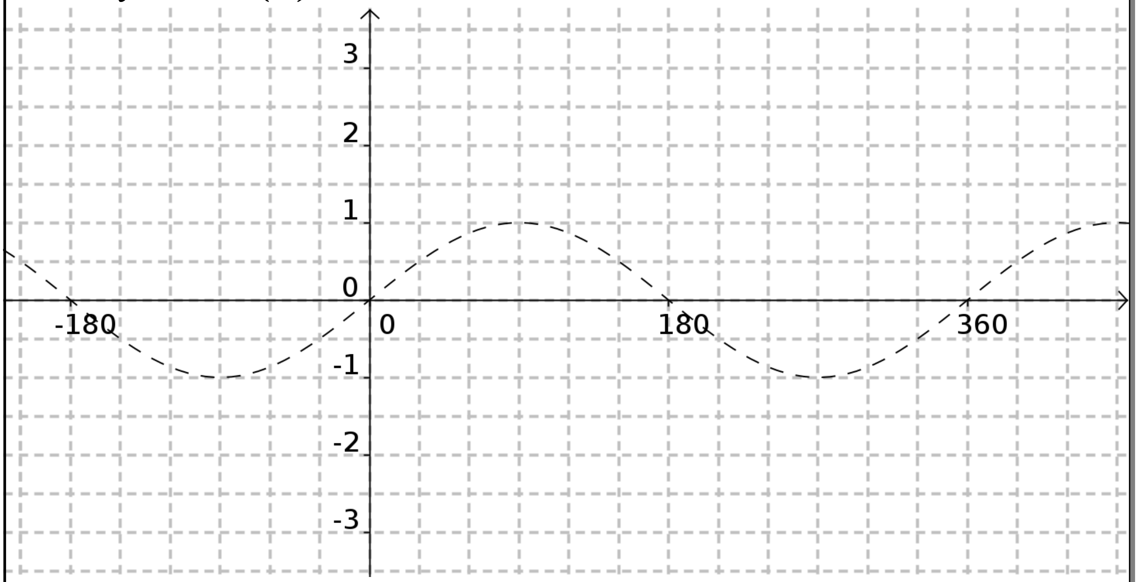


$k = 3 \longrightarrow$ h. compression by 3 $\quad \frac{x}{3}$

$(0°, 1) \longrightarrow (0°, 1)$
$(90°, 0) \longrightarrow (30°, 0)$
$(180°, -1) \longrightarrow (60°, -1)$
$(270°, 0) \longrightarrow (90°, 0)$
$(360°, 1) \longrightarrow (120°, 1)$

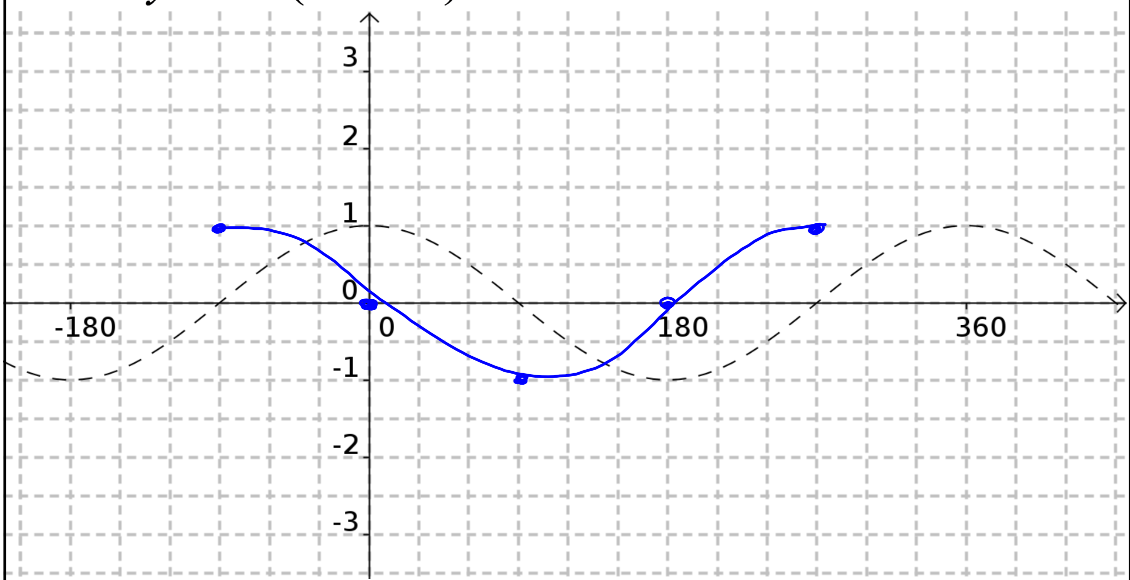
Dec 6-11:03 AM

(c) $y = \sin(x) + 2$



Dec 6-11:04 AM

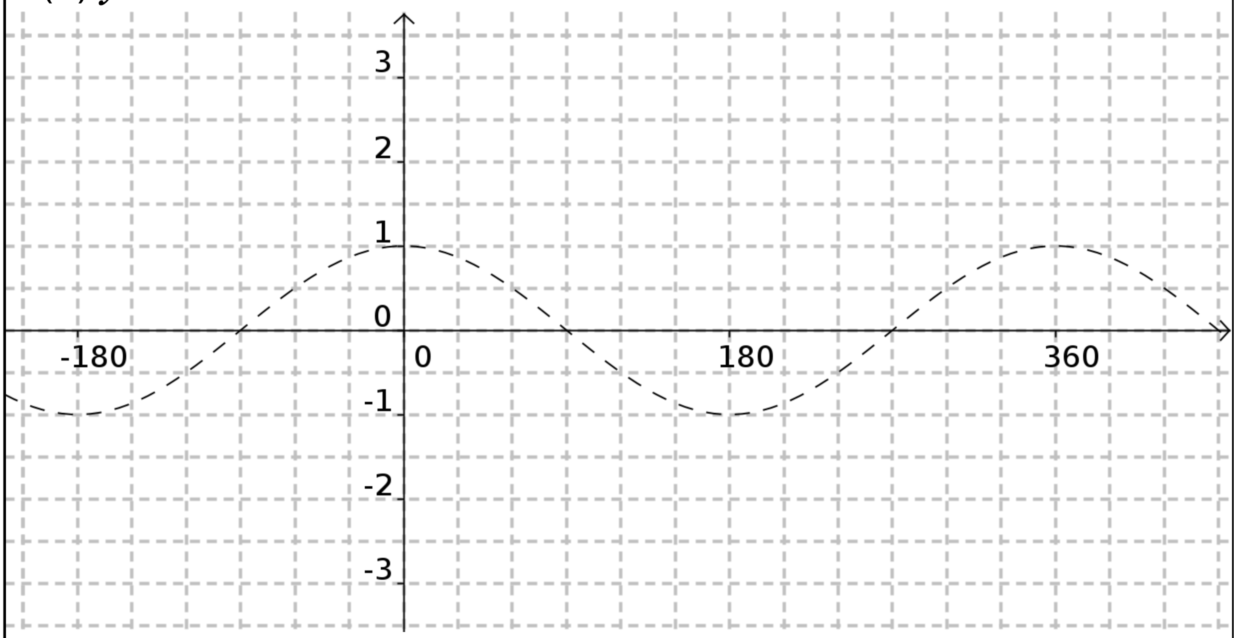
(d) $y = \cos(x + 90°)$



$P = -90°$ → h. left by 90°

Dec 6-11:04 AM

(e) $y = -\cos x$



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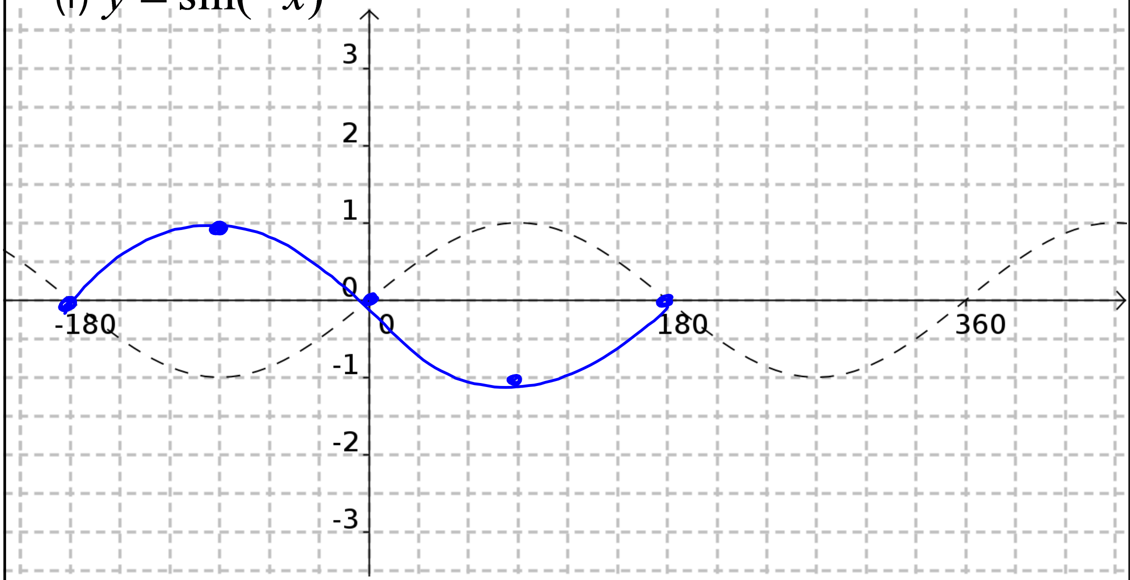
(f) $y = \sin(-x)$



$k = -1 \longrightarrow$ h. reflection $\quad x \longrightarrow -x$

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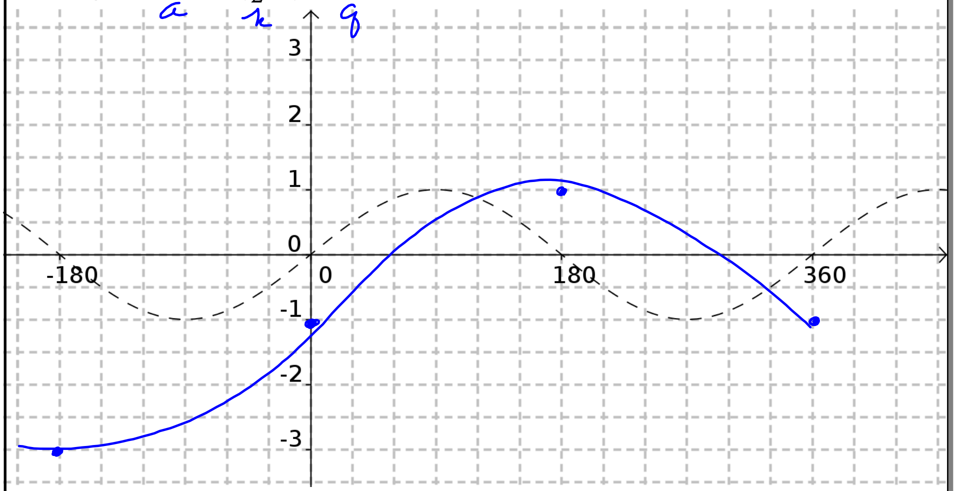
(g) $y = 2\sin(\frac{1}{2}x) - 1$

a k q



V. stretch by 2
h. stretch by 2
V. down by 1

$(x, y) \rightarrow (2x, 2y - 1)$

$(0, 0) \rightarrow (0, -1)$

$(90°, 1) \rightarrow (180°, 1)$

$(-90°, -1) \rightarrow (-180°, -3)$

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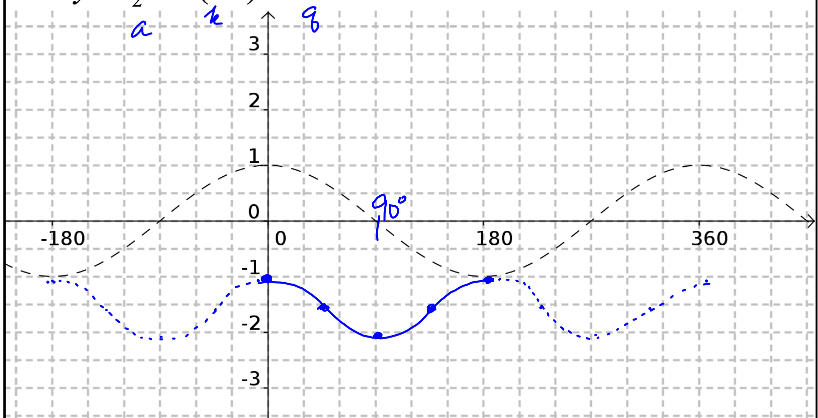
(h) $y = \frac{1}{2}\cos(2x) - 1.5$

a k q



① V. compress by 2
② h. compress by 2
③ V. down by 1.5

$(x, y) \rightarrow (\frac{x}{2}, \frac{y}{2} - 1.5)$

$(0°, 1) \rightarrow (0°, -1)$

$(90°, 0) \rightarrow (45°, -1.5)$

$(180°, -1) \rightarrow (90°, -2)$

$(270°, 0) \rightarrow (135°, -1.5)$

$(360°, 1) \rightarrow (180°, -1)$

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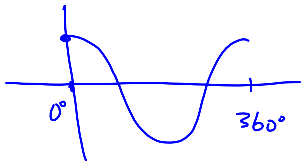
p.374 # 1ad, 3ade, 6abe
p.387 # 1abceh, 3ad

p.374 3(d) $y = \cos\left(\frac{1}{4}x\right)$

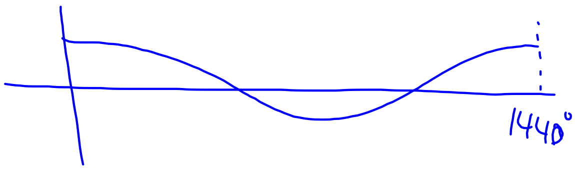
one cycle
$y = \cos x$



$D = \{x \in \mathbb{R} \mid 0° \leq x \leq 360°\}$

one cycle
of $y = \cos\left(\frac{1}{4}x\right)$

h. stretch by 4



$D = \{x \in \mathbb{R} \mid 0° \leq x \leq 1440°\}$

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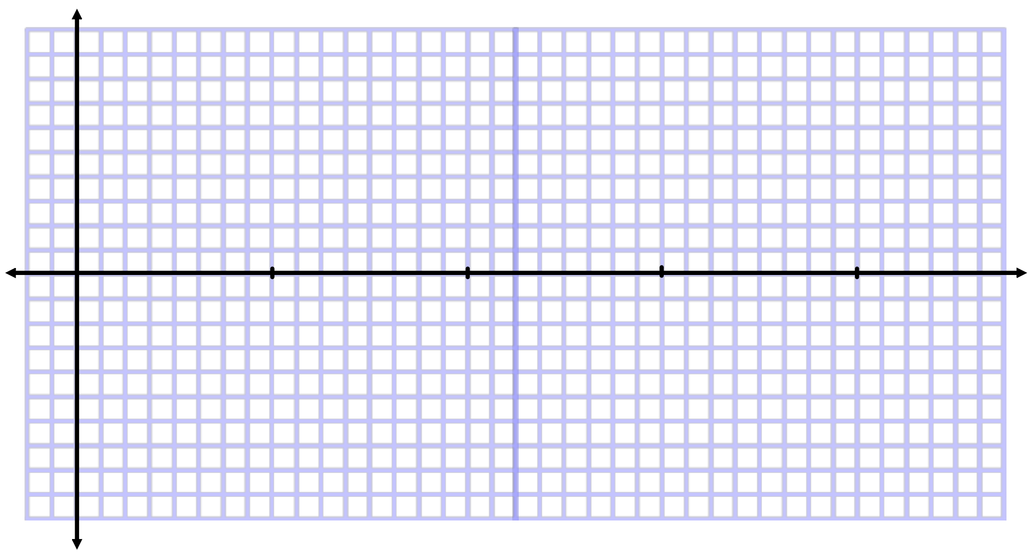

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p. 387 #1(e)

$$y = \sin(\underset{p}{\overset{x-p}{x - 60^\circ}}) + \underset{q}{1}$$

① h. right by 60° → phase shift of +60°

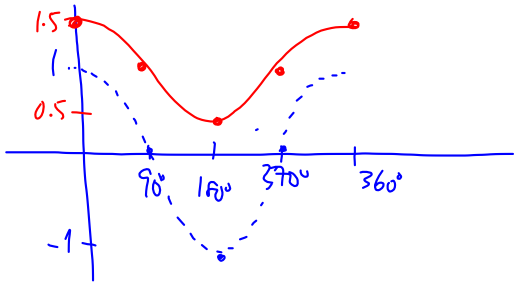
② v. up by 1

May 22-2:09 PM

p. 387 3(d) $y = \underset{a}{\frac{1}{2}} \cos x + \underset{q}{1}$

① v. compress by 2 $\frac{y}{2}$

② v. shift up by 1 $y + 1$



amplitude = 0.5

period = 360°

$D = \{x \in \mathbb{R} \mid 0^\circ \le x \le 360^\circ\}$

$R = \{y \in \mathbb{R} \mid 0.5 \le y \le 1.5\}$

May 22-2:11 PM