

Unit 7: Discrete Functions
Arithmetic & Geometric Sequences

June 5/2019

Ex.1 Find the next 3 terms in each sequence:

(a) 3, 7, 11, 15, ... , 19, 23, 27
 $\begin{array}{c} \uparrow \uparrow \uparrow \\ +4 +4 +4 \end{array}$

(b) 9, 4, -1, -6, ... -11, -16, -21
 $\begin{array}{c} \uparrow \uparrow \uparrow \\ -5 -5 -5 \end{array}$

(c) 1, 1.25, 1.5, 1.75, ... 2.00, 2.25, 2.50
 $\begin{array}{c} \uparrow \uparrow \uparrow \\ +0.25 +0.25 \end{array}$

May 27-2:43 PM

Ex.2 For the sequence 3, 9, 15, 21, ... 27, 33, 39

(a) find the 100th term. $\begin{array}{c} \uparrow \uparrow \\ +6 +6 \end{array}$

(b) find a general expression for the nth term.

$$(a) t_{100} = 3 + \underline{99}(6)$$

$$= 597$$

$$(b) t_n = 3 + (n-1)(6) \quad \checkmark$$

OR, expand

$$t_n = 3 + 6n - 6$$

$$t_n = 6n - 3 \quad \checkmark$$

May 29-4:20 PM

An arithmetic sequence is a *linear function* where the difference between consecutive terms is a constant (called the common difference, d).

The first term, t_1 , or $f(1)$, is a .

In general, the sequence is:

$$a, a+d, a+2d, a+3d, \dots$$

The n^{th} term is:

$$t_n = a + (n-1)d \quad \text{or} \quad f(n) = a + (n-1)d$$

May 28-9:27 PM

Ex.3 How many terms are in the finite sequence

$$\begin{aligned}
 & \text{16, } 7, -2, -11, \dots, -245? \\
 & a = 16, d = -9 \\
 & t_n = a + (n-1)d \\
 & t_n = 16 + (n-1)(-9) \\
 & \text{OR} \\
 & t_n = 16 - 9(n-1) \\
 & -245 = 16 - 9(n-1) \\
 & -261 = -9(n-1) \\
 & \frac{-261}{-9} = \frac{-9(n-1)}{-9} \\
 & 29 = n-1 \\
 & 30 = n
 \end{aligned}$$

May 29-4:23 PM

Ex.4 Find the next three terms in each sequence:

(a) 2, 4, 8, 16, ... $\begin{matrix} \text{32, } 64, \\ \curvearrowleft \curvearrowleft \\ \times 2 \end{matrix}$

(b) 1, -2, 4, -8, ... $\begin{matrix} 16, -32, 64 \\ \curvearrowleft \curvearrowleft \\ \times (-2) \end{matrix}$

(c) 27, 9, 3, 1, ... $\begin{matrix} \frac{1}{3}, \frac{1}{9}, \frac{1}{27} \\ \curvearrowleft \\ \times \frac{1}{3} \end{matrix}$

May 29-4:25 PM

Ex.5 For the sequence 5, 10, 20, 40, ...

- (a) find the 8th term.
- (b) write an expression for the nth term.
- (c) where would you find 5120 in the sequence?

(a) 5, 10, 20, 40, $\underline{80}$, $\underline{160}$, $\underline{320}$, $\underline{640}$
 1 2 3 4 5 6 7 8

$$t_8 = 640$$

(b) $t_1 = 5$ $t_n = a + (n-1)d$

$$t_2 = 5 \cdot 2$$

$$t_8 = 5 \cdot 2^7$$

$$t_n = 5 \cdot 2^{n-1}$$

(c) $5120 = 5(2^{n-1})$

$$1024 = 2^{n-1}$$

$$2^{10} = 2^{n-1}$$

$$\Rightarrow 10 = n-1 \\ n = 11$$

May 29-4:27 PM

A geometric sequence occurs when there is a common ratio (r) between consecutive terms.

The first term, t_1 , or $f(1)$, is a .

In general, the sequence is:

$$a, \underset{1}{ar}, \underset{2}{ar^2}, ar^3, \dots$$

The n^{th} term is:

$$t_n = ar^{n-1} \quad \text{or} \quad f(n) = ar^{n-1}$$

May 29-4:29 PM

Ex.6 Is each sequence geometric? If so, state the common ratio.

(a) $2, -8, 32, -128, \dots$ $\times (-4) \quad \times (-4) \quad \times (-4)$ OR $\frac{t_2}{t_1} = -4 \quad \frac{t_3}{t_2} = -4 \quad \frac{t_4}{t_3} = -4$

\checkmark geometric

(b) $x, 2x, 3x, 4x, \dots$ $\times 2 \quad \times 1.5$ OR $\frac{2x}{x} = 2 \quad \frac{3x}{2x} = \frac{3}{2} \quad \frac{4x}{3x} = \frac{4}{3}$

(c) $x^7, x^{14}, x^{28}, x^{56}, \dots$ \times ratios different

$$\frac{x^{14}}{x^7} = x^7 \quad \frac{x^{28}}{x^{14}} = x^{14} \quad \times \text{ratios are different}$$

(d) $2x^7, 4x^{10}, 8x^{13}, 16x^{16}, \dots$

$$\frac{4x^{10}}{2x^7} = 2x^3 \quad \frac{8x^{13}}{4x^{10}} = 2x^3 \quad \frac{16x^{16}}{8x^{13}} = 2x^3$$

\checkmark geometric, common ratio $r = 2x^3$

May 29-4:34 PM

Ex.7 Given $t_5 = 1875$ and $t_7 = 46875$, find t_n (geometric).

$$t_n = ar^{n-1}$$

$$t_5 = 1875 \quad t_7 = 46875$$

$$\begin{array}{rcl} 1875 = ar^4 & \textcircled{1} & 46875 = ar^6 & \textcircled{2} \\ & & 1875 = ar^4 & \textcircled{1} \\ \hline & & & \end{array}$$

$$\textcircled{2} \div \textcircled{1} \quad 25 = r^2$$

$$r = \pm 5$$

$$\frac{r = 5}{t_n = a(5)^{n-1}}$$

$$1875 = a(5)^4$$

$$a = 3$$

May 29-4:38 PM

Assigned Work:

$$\begin{array}{rcl} 4i & \frac{7g}{8a} \\ p.441 \# [1-4][\text{basics}], 5-8(\text{adj}), 9) 12, 15, 18 & \text{A} \\ p.452 \# [1-4][\text{basics}], 5-7(\text{adf}), 9, 13, 18 \end{array}$$

$$7d$$

$$5d$$

$$\begin{array}{l} P.441 \quad 4(i) \quad x, 2x, 3x, 4x \\ a=x \quad \nearrow \quad \curvearrowright \quad \curvearrowright \\ +x \quad +x \quad +x \quad \rightarrow \text{arithmetic} \\ d=x \end{array}$$

$$t_n = a + (n-1)d$$

$$= x + (n-1)x \quad \checkmark$$

$$= x + nx - x$$

$$t_n = nx \quad \checkmark$$

May 27-3:05 PM

p.441 7(g)

$$\begin{array}{c} x+2, x+9, x+16, \dots, x+303 \\ \swarrow \quad \searrow \\ +7 \qquad +7 \\ d=7 \end{array}$$

$$a=x+2$$

$$t_n = a + (n-1)d$$

$$t_n = (x+2) + (n-1)(7)$$

$$x+303 = x+2 + [7(n-1)]$$

$$\frac{301}{7} = \frac{7(n-1)}{7}$$

$$43 = n-1$$

$$n=44$$

$$x+303 = x+2 + 7n - 7$$

$$308 = 7n$$

$$n=44$$

Jun 6-1:58 PM

p. 441 8(a) $t_5 = 16 \quad t_8 = 25$

$$\begin{array}{c} \text{---} \\ | \qquad \qquad \qquad \uparrow \\ 3 \text{ steps of } d = 9 \end{array}$$

$$3d = 9$$

$$t_n = a + (n-1)d \quad d = 3$$

$$16 = a + 4d \quad 25 = a + 7d$$

$$16 = a + 4d \quad \textcircled{1}$$

$$\textcircled{2} - \textcircled{1}: \quad 9 = 3d$$

$$d = 3$$

Sub $d = 3$ into $\textcircled{1}$ or $\textcircled{2}$

$$16 = a + 4(3)$$

$$16 = a + 12$$

$$a = 4$$

$$\therefore t_n = 4 + (n-1)(3)$$

$$= 4 + 3(n-1) \checkmark$$

$$= 4 + 3n - 3$$

$$= 1 + 3n \checkmark$$

Jun 6-2:03 PM

$$\text{p. 441 #9, } t_3 = 24 \quad t_9 = 54$$

$$t_n = a + (n-1)d$$

$$24 = a + 2d \quad ① \quad 54 = a + 8d \quad ②$$

Jun 6-2:09 PM

P. geo 5d, 7d

$$\begin{array}{l} 5(d) \quad 64, 32, 16, \dots \quad t_7, t_{10} \\ \text{---} \\ a = 64 \quad \underbrace{\times \frac{1}{2} \quad \times \frac{1}{2}}_{r = \frac{1}{2}} \quad \begin{array}{l} t_n = ar^{n-1} \\ t_n = 64 \left(\frac{1}{2}\right)^{n-1} \end{array} \end{array}$$

$$\begin{array}{ll} t_7 = 64 \left(\frac{1}{2}\right)^{7-1} & t_{10} = 64 \left(\frac{1}{2}\right)^9 \\ = 64 \left(\frac{1}{2}\right)^6 & = 64 \left(\frac{1}{2^9}\right) \\ = 64 \left(\frac{1}{64}\right) & = 64 \left(\frac{1}{512}\right) \\ = 1 & = \frac{1}{8} \end{array}$$

Jun 6-2:11 PM

$$p. 453 \quad 7(d) \quad t_2 = 4 \quad t_4 = 64$$

$$t_n = ar^{n-1}$$

$$\textcircled{1} \quad 4 = ar^1 \quad \textcircled{2} \quad 64 = ar^3$$

$$a = \frac{4}{r}$$

Sub

$$64 = ar^3 \quad \textcircled{2}$$

$$4 = ar \quad \textcircled{1}$$

$$\frac{\textcircled{2}}{\textcircled{1}} \quad 16 = r^2$$

$$64 = \left(\frac{4}{r}\right)r^3$$

$$64 = 4r^2$$

$$16 = r^2$$

$$r = \pm 4$$

$$r = 4$$

$$r = -4$$

$$4 = a(4)$$

$$4 = a(-4)$$

$$a = 1$$

$$a = -1$$

$$t_n = (1)(4)^{n-1} \quad \text{or} \quad t_n = (-1)(-4)^{n-1}$$

$$t_n = 4^{n-1}$$

Jun 6-2:15 PM