Unit 7: Discrete Functions
Geometric Sequences

Ex. Find the next three terms in each sequence:

(a) 2, 4, 8, 16, ... 32, 64, 128

×2 ×2 ×2

(b) 1, -2, 4, -8, ... 16, -32, 64

×(-2) ×(-2)

(c) 27, 9, 3, 1, ... $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$

÷3 ÷3 ÷3

$\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$

y	$\frac{y_2}{y_1}$
2	
4	2
8	2
16	2
32	2

common ratio = 2

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Ex. For the sequence 5, 10, 20, 40, ... 80, 160, 320, 640

(a)

(a) find the 8th term.
(b) write an expression for the nth term.
(c) where would you find 5120 in the sequence?

recall: arithmetic sequence $t_n = a + (n-1)d$

start, # of steps, step

(b) $t_1 = 5$, $t_2 = 10 = 5(2)^1$ (×2)

$t_1 = 5$, $t_3 = 20 = 5(2)(2) = 5(2)^2$ (×2×2 = 2^2)

$t_1 = 5$, $t_4 = 40 = 5(2)(2)(2) = 5(2)^3$

$$\boxed{t_n = 5(2)^{n-1}}$$

(c) $t_n = 5120$, $n = ?$

$\frac{5120}{5} = \frac{5[(2)^{n-1}]}{5}$

$1024 = 2^{n-1}$

$2^{10} = 2^{n-1}$

$\Rightarrow 10 = n-1$

$n = 11$

2	1
4	2
8	3
16	4
32	5
64	6
128	7
256	8
512	9
1024	10

∴ 5120 is term number 11.

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A <u>geometric sequence</u> occurs when there is a <u>common ratio</u> (r) between consecutive terms.

The first term, t_1, or $f(1)$, is a.

In general, the sequence is:

$$\underset{t_1}{a},\ \underset{t_2}{ar^1},\ \underset{t_3}{ar^2},\ \underset{t_4}{ar^3},\ \ldots$$

The n^{th} term is:

$$t_n = ar^{n-1} \quad \text{or} \quad f(n) = ar^{n-1}$$

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Ex. Is each sequence geometric? If so, state the common ratio.

(a) $\underset{t_1}{2}, \underset{t_2}{-8}, \underset{t_3}{32}, \underset{t_4}{-128}, \ldots$

$\frac{t_2}{t_1} = \frac{-8}{2} = -4 \quad \frac{t_3}{t_2} = \frac{32}{-8} = -4 \quad \frac{t_4}{t_3} = \frac{-128}{32} = -4$

✓ geometric, $r = -4$

(b) $\underset{t_1}{x}, \underset{t_2}{2x}, \underset{t_3}{3x}, \underset{t_4}{4x}, \ldots$

$\frac{t_2}{t_1} = \frac{2x}{x} = 2 \quad \frac{t_3}{t_2} = \frac{3x}{2x} = \frac{3}{2}$

$2 \neq \frac{3}{2}$

x not geometric

(c) $\underset{t_1}{x^7}, \underset{t_2}{x^{14}}, \underset{t_3}{x^{28}}, \underset{t_4}{x^{56}}, \ldots$

$\frac{t_2}{t_1} = \frac{x^{14}}{x^7} = x^7 \quad \frac{t_3}{t_2} = \frac{x^{28}}{x^{14}} = x^{14}$

x not geo.

(d) $2x^7, 4x^{10}, 8x^{13}, 16x^{16}, \ldots\ 32x^{19}$

$\frac{t_2}{t_1} = \frac{4x^{10}}{2x^7} = 2x^3 \quad \frac{t_3}{t_2} = \frac{8x^{13}}{4x^{10}} = 2x^3 \quad \frac{16x^{16}}{8x^{13}} = 2x^3$

✓ geo

$r = 2x^3$

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Ex. Given $t_5 = 1875$ and $t_7 = 46875$, find t_n (geometric).

$t_n = ar^{n-1}$

$1875 = ar^4$ ①

$46875 = ar^6$ ②

$1875 = ar^4$ ①

② ÷ ① $25 = r^2$

$r = \pm 5$

① ⇒ $a = \frac{1875}{r^4}$

sub a into ②

$46875 = \left(\frac{1875}{r^4}\right) r^6$

$\frac{46875}{1875} = \frac{1875 r^2}{1875}$

$25 = r^2$

$r = 5$

① $1875 = a(5)^4$

$1875 = a(625)$

$a = 3$

$r = -5$

$1875 = a(-5)^4$

$1875 = a(625)$

$a = 3$

$\therefore t_n = 3(5)^{n-1}$

OR

$t_n = 3(-5)^{n-1}$

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Assigned Work:

p.452 # [1-4][basics], 5-7(adf), 9, 13, 18

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