Recursion Formulae & Recursive Sequences

Determine the pattern in the Fibonacci sequence:

1, 1, 2, 3, 5, 8, ...

$t_1 = 1$ ✓

$t_2 = 1$ ✓

$t_3 = t_2 + t_1$
$= 1 + 1$
$= 2$

$t_4 = t_3 + t_2$
$= 2 + 1$
$= 3$

$t_n = t_{n-1} + t_{n-2}$

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Recursion Formulae & Recursive Sequences

June 6/2019

A sequence is recursive if a new term is found using a previous term (or terms).

Ex.1 Find the first three terms in each of the following sequences

a) $t_n = t_{n-1} - 2$ where $t_1 = 3$ $\quad n \in \mathbb{N}$

$t_2 = t_1 - 2$
$= 1$

$t_3 = t_2 - 2$
$= -1$

$t_4 = t_3 - 2$
$= -3$

b) f(n) = f(n - 1) + 1.5 where f(1) = - 0.5

$f(1) = -0.5$

$f(2) = f(1) + 1.5$
$= -0.5 + 1.5$
$= 1$

$f(3) = f(2) + 1.5$
$= 2.5$

$f(4) = f(3) + 1.5$
$= 4$

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Ex.2 Determine a recursion formula for each of the following sequences, then write an explicit formula if possible (an explicit formula does not rely on recursion).

a) -3, 6, -12, 24, ... ×(-2)

$t_n = -2t_{n-1}$, where $t_1 = -3$ recursive

OR

$t_n = -3(-2)^{n-1}$ geometric

b) f(1) = 2, f(2) = 6, f(3) = 10, f(4) = 14, ...

+4 +4 +4

$f(n) = f(n-1) + 4$, where $f(1) = 2$ recursive

OR $f(n) = 2 + (n-1)(4)$ arithmetic

c) 3, 5, 8, 12, ... 17, 23

+2 +3 +4 +5

+1 +1 +1, $2a = 1$

$t_n = an^2 + bn + c$

$t_n = \frac{1}{2}n^2 + bn + c$

$t_2 = t_1 + 2$

$t_3 = t_2 + 3$

$t_4 = t_3 + 4$

$t_{100} = t_{99} + 100$

$t_n = t_{n-1} + n$ recursive

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Pascal's Triangle & The Binomial Theorem

Consider:

$(x+y)^0 = 1$

$(x+y)^1 = 1x + 1y$

$(x+y)^2 = 1x^2 + 2xy + 1y^2$

$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$

$1x^4 + 4x^3y + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$

1 5 10 10 5 1

$(x+y)^7 =$

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Pascal's Triangle & The Binomial Theorem

The coefficients from the *binomial expansions* can be organized as a triangle, where each number in the triangle is the sum of the two numbers above it.

$(x+y)^7 =$

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Ex.3 Use Pascal's Triangle to expand $(x - 3)^3$.

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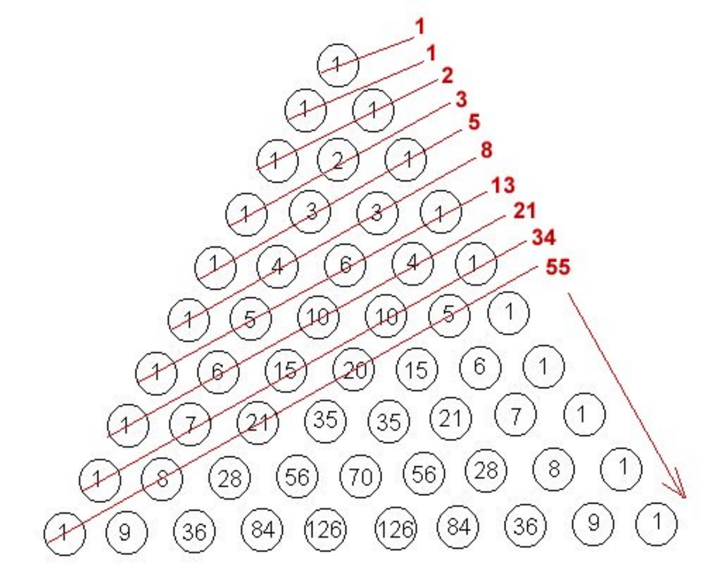
Fibonacci Numbers from Pascal's Triangle

The Fibonacci numbers can be obtained by adding the *shallow diagonals* in Pascal's triangle.



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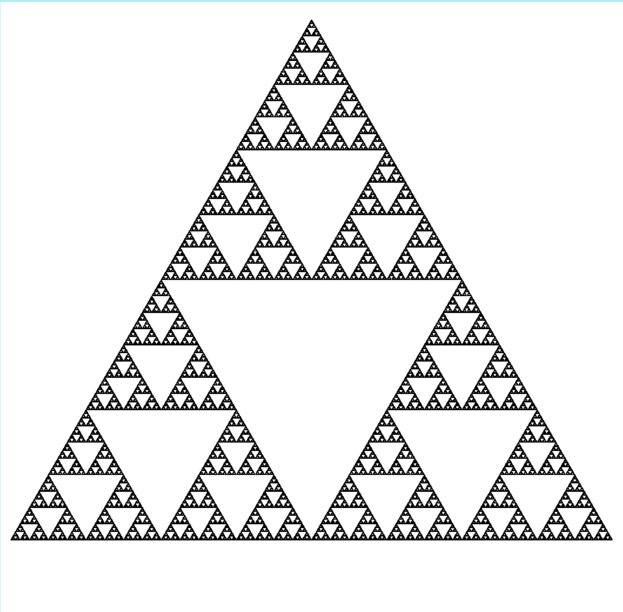
Recursion can also be applied to geometric figures to produce **fractals**, patterns that repeat themselves at any scale (zooming in or out).

One of the simplest and most famous is the Serpenski Triangle:



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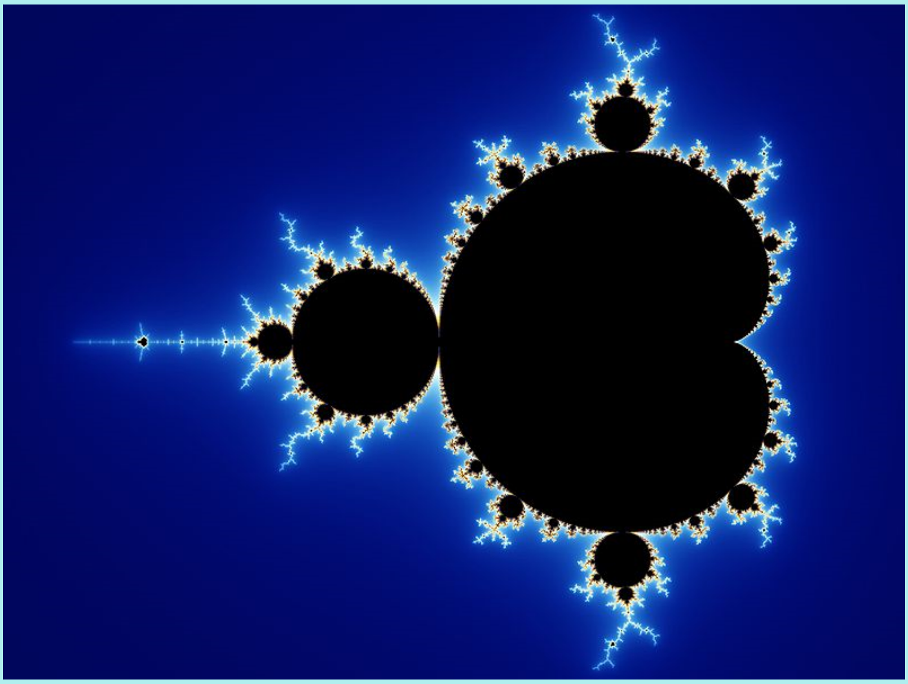
The Mandelbrot Set: A recursively defined set of points



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Assigned Work:

p.461 #[1-5] basics, 8, 9, 10, 12

~~Expand $(a - b)^5$, $(2x + 5y)^4$~~

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