

Applications of Sequences:

Simple & Compound Interest

June 10/2019

Simple interest is only paid on the principal (initial investment) and can be modelled as an arithmetic sequence.

Many GICs (Guaranteed Investment Credits) calculate interest in this way.

Ex.1 \$2000 is invested in a 5-year GIC that pays 2.1% per year, calculated annually. What is the value of the investment when it matures?

$$y_1: \text{interest} = 2000(0.021) = 42$$

$$2.1\% = \frac{2.1}{100} = 0.021$$

$$y_2: 2000(0.021) = 42$$

$$y_3: 42$$

$$y_4: 42$$

$$y_5: 42$$

$$5(42) = 210$$

$$A = \underbrace{2000}_{\text{principal}} + \underbrace{2000(0.021)(5)}_{\text{interest}}$$

$$A = 2210$$

\therefore the GIC is worth \$2210 after 5 years.

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In general, for simple interest:

$$A = P + Prt \quad \text{where}$$

- A is the final amount
- P is the initial investment (or principal)
- r is the rate of interest
- t is the term (time)

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Compound interest is earned on both the principal as well as any interest earned as the investment grows. This can be modelled as a geometric sequence.

It is now important to consider the compounding period, which is how frequently interest is calculated, as well as the rate of interest (usually quoted per annum, or year).

Savings accounts will generally pay this type of interest.

Ex.2 \$5000 is invested for 10 years at 1.5%, ~~0.015~~ compounded annually. What is the final value?

interest

$$y1: 5000(0.015) = 75$$

$$y2: 5075(0.015) = 76.125$$

$$y3: 5151.125(0.015) = \underline{\quad}$$

final value $100\% + 1.5\% = 101.5\%$
 $= 1.015$

$$y1: 5000 \begin{matrix} \nearrow 100\% \\ \searrow 1.5\% \text{ interest} \end{matrix} (1.015) = 5075$$

$$y2: 5075(1.015) = 5000(1.015)(1.015)$$

$$= 5000(1.015)^2$$

$$y3: 5000(1.015)^2(1.015)$$

$$= 5000(1.015)^3$$

$$y_{10}: 5000(1.015)^{10} = 5802.70$$

\therefore after 10 years, investment is worth \$5802.70

$$1000(1.14)^{30} = 50950.16$$

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In general, for compound interest:

$$A = P(1 + r)^n \quad \text{where}$$

- A is the final amount
- P is the principal invested
- r is the rate of interest
- per compounding period**
- n is the number of compounding periods

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Ex.3 Compare the following investment options:

(a) 9 years at 6% per annum, compounded semi-annually

(b) 9 years at 5.95% per annum, compounded monthly

per year

assume $P = \$1000$

(a) $t = 9$ years $i = 6\%/year$

semi-annually
(2x per year)

$$n = 9 \times 2 \\ = 18$$

$$r = \frac{6\%}{2}$$

$$= 3\%$$

$$= 0.03$$

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Assigned Work:

p.508 # 2 - 5, 7, 11, 13

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