

Transformations of Functions see handout

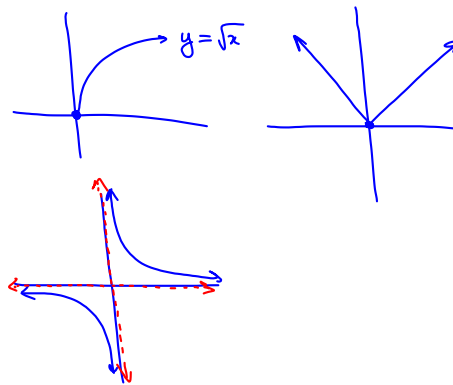
Recall: In Gr.10, we explored transformations of a quadratic relation. For consistency, we use vertex form.

$$y = a(x - h)^2 + k$$

What are the possible transformations?
(read from left to right)

① $a \rightarrow$ v. reflection
 \rightarrow v. scaling
 \rightarrow stretch
 \rightarrow compression

② $h \rightarrow$ h. shift } $\Rightarrow V(h, k)$
 ③ $k \rightarrow$ v. shift }



$$y = a(x - h)^2 + k$$

1&2
3
4

Summary of possible transformations (in order):

1. Vertical Reflection: $a < 0$
2. Vertical Scaling (stretch or compress):
 - stretch when $a < -1$ or $a > 1$ (or $|a| > 1$)
 - parabola is thinner
 - compress when $-1 < a < 1$ (or $|a| < 1$)
 - parabola is wider
3. Horizontal Translation (shift left or right) by h
4. Vertical Translation (shift up or down) by k

Using function notation,

$$y = x^2 \text{ becomes } y = f(x)$$

$$\text{so } y = a(x-h)^2 + k \text{ becomes } y = af(x-h) + k$$

The transformations produced by a, h, and k can be applied to any function.

Start with:

$$f(x) = x^2$$

$$f(x-h) = (x-h)^2$$

$$af(x-h) = a(x-h)^2$$

$$af(x-h) + k = a(x-h)^2 + k$$

Ex.1 State the transformations, in the correct order, that $y = f(x)$ has undergone to obtain $y = f(x+1) + 3$

① h. shift left by 1

② v. shift up by 3

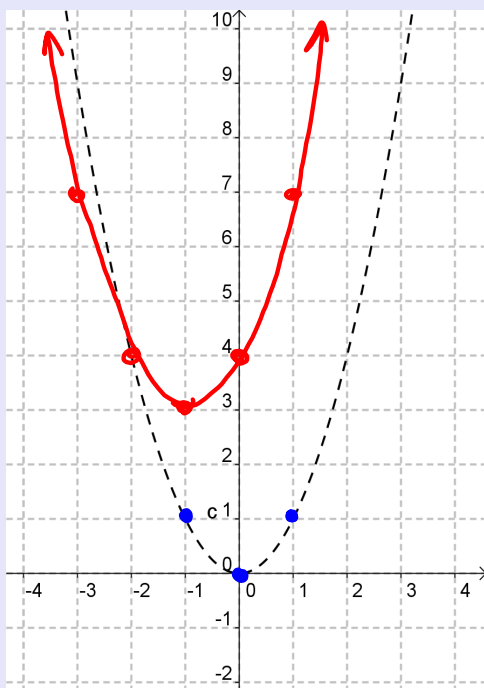
$$(x, y) \rightarrow (x-1, y+3)$$

x	$y = x^2$	$y = (x-1)^2$
-3	9	16
-2	4	9
-1	1	4
0	0	1
1	1	0
2	4	1
3	9	4

Using the previous example, what if the parent function is a quadratic? Then $f(x) = x^2$.

Sketch the transformed parabola on the grid below.

(Use a table of values if you want to)



$$y = 1f(x+1) + 3$$

① left by 1

② up 3

$$f(x) = x^2$$

$$f(x+1) = (x+1)^2$$

$$f(x+1) + 3 = (x+1)^2 + 3$$

$$y = (x+1)^2 + 3$$

What if the parent function is a set of ordered pairs?

If $f(x) = \{(1, 2), (2, -3), (-5, 7)\}$

$$y = f(x)$$

what would $y = f(x+1) + 3$ become?

(Use a table of values and/or graph if you want to!)

parabola: $y = (x+1)^2 + 3$

$$V(0,0) \rightarrow V(-1,3)$$

$$(x,y) \rightarrow (x-1, y+3)$$

$$(1,2) \rightarrow (0,5)$$

$$(2,-3) \rightarrow (1,0)$$

$$(-5,7) \rightarrow (-6,10)$$

What if the parent function is the square root function?

Then $f(x) = \sqrt{x}$.

What would $y = f(x+1) + 3$ become?

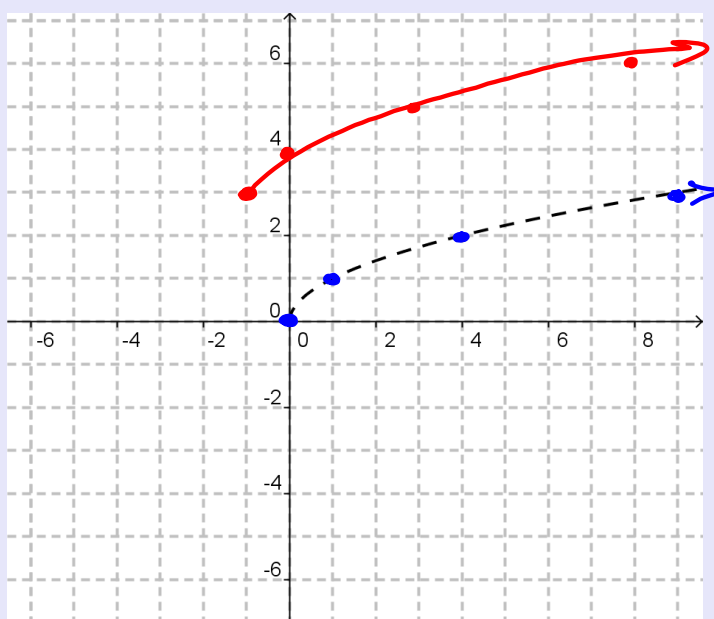
(Use a table of values if you want to)

$$(0,0) \rightarrow (-1,3)$$

$$(1,1) \rightarrow (0,4)$$

$$(4,2) \rightarrow (3,5)$$

$$(9,3) \rightarrow (8,6)$$



What if the parent function is the reciprocal function?

Then $f(x) = \frac{1}{x}$.

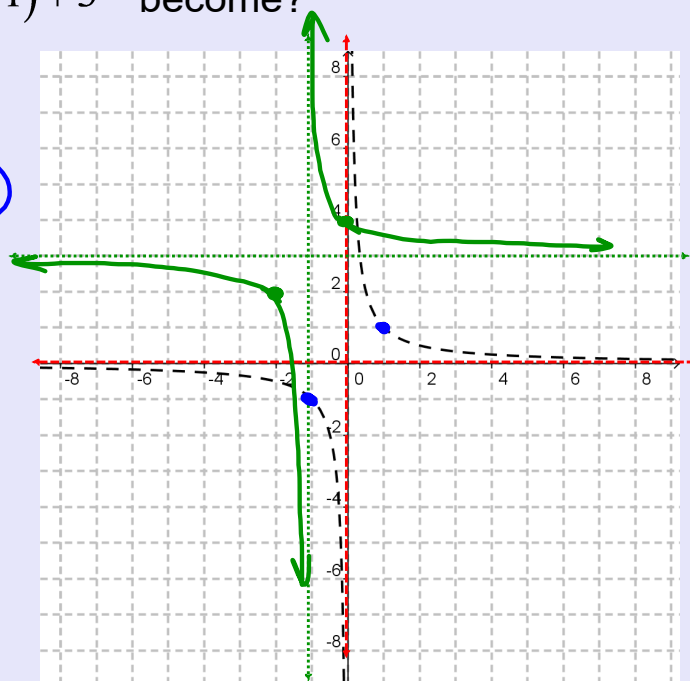
What would $y = f(x+1) + 3$ become?

(Use a table of values if you want to)

VA and HA cross
at $(0,0) \rightarrow (-1,3)$

$$(1,1) \rightarrow (0,4)$$

$$(-1,-1) \rightarrow (-2,2)$$



Ex: Given $y = 2f(x) + 3$ describe the transformations and apply them to the square root function.

(Use a table of values and/or graph if you want to!)

for parabola

$$f(x) = x^2$$

$$2f(x) + 3 = 2x^2 + 3$$

a k

① v. stretch by 2
 $y \rightarrow 2y$

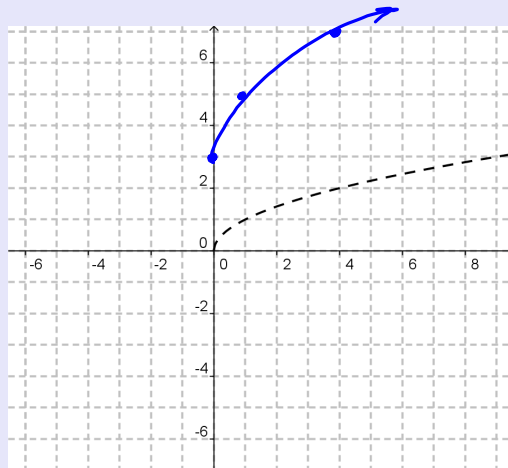
② v. shift up by 3
 $y \rightarrow y + 3$

$$(x, y) \rightarrow (x, 2y + 3)$$

$$(0, 0) \rightarrow (0, 3)$$

$$(1, 1) \rightarrow (1, 5)$$

$$(4, 2) \rightarrow (4, 7)$$



$$f(x) = \sqrt{x}$$

$$2f(x) = 2\sqrt{x}$$

$$2f(x) + 3 = 2\sqrt{x} + 3$$

Ex: Given $y = -3f(x-1)$, describe the transformations and apply them to the reciprocal function.

(Use a table of values and/or graph if you want to!)

① v. reflect

② v. stretch by 3

③ h. shift right 1

$$(x, y) \rightarrow (x+1, -3y)$$

$$(0, 0) \rightarrow (1, 0)$$

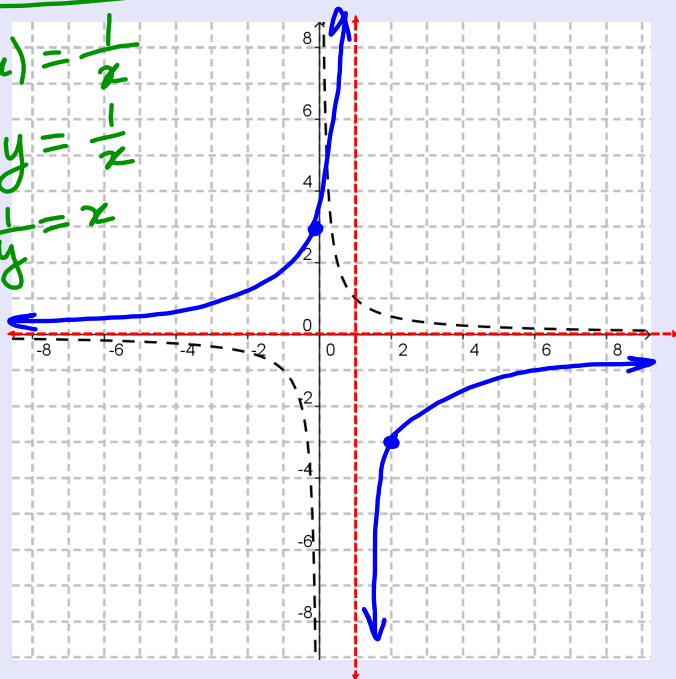
$$(1, 1) \rightarrow (2, -3)$$

$$(-1, -1) \rightarrow (0, 3)$$

$$f(x) = \frac{1}{x}$$

$$y = \frac{1}{x}$$

$$\frac{1}{y} = x$$



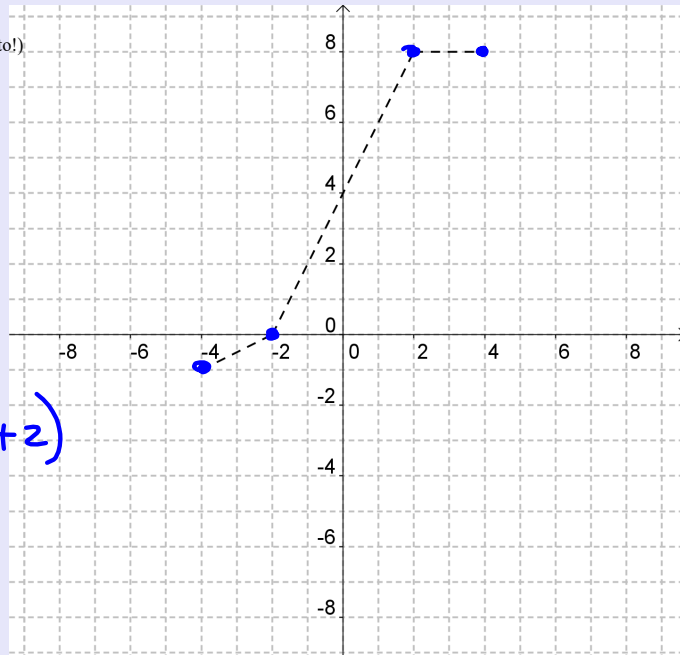
Ex: The graph on the right shows the function $y = f(x)$.
 Given $y = -f(x+1) + 2$, describe the transformations and apply them to the function

(Use a table of values with the key points, if you want to!)

- ① v. reflect
- ② h. shift left 1
- ③ v. shift up 2

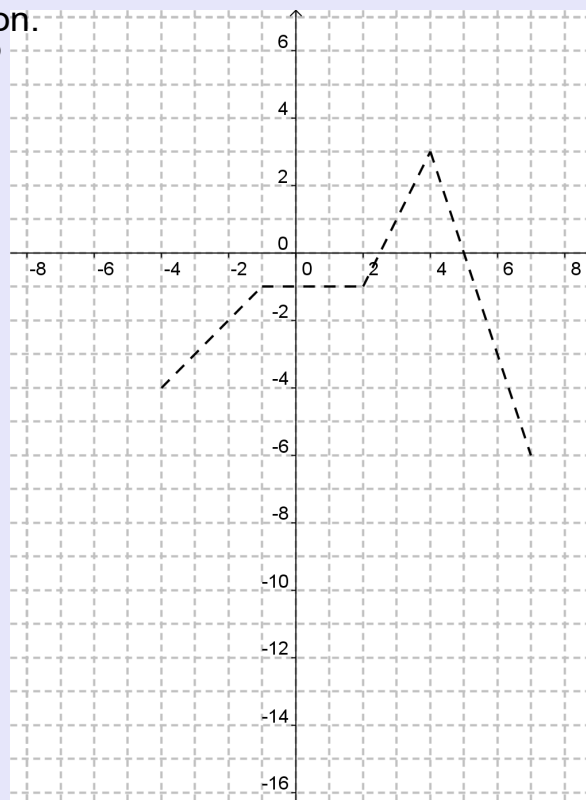
$$(x, y) \rightarrow (x-1, -y+2)$$

$$(-4, -1) \rightarrow (-5, 3)$$



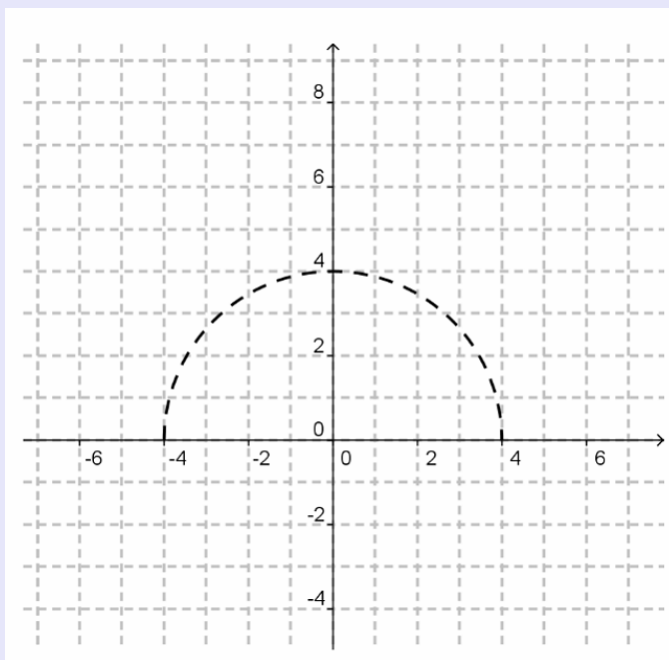
Ex: The graph on the right shows the function $y = f(x)$.
 Given $y = 2f(x) - 1$, describe the transformations and apply them to the function.

(Use a table of values with the key points, if you want to!)



Ex: The graph on the right shows the function $f(x) = \sqrt{16 - x^2}$. Given $y = -0.5f(x+2)$, describe the transformations and apply them to the function.

(Use a table of values with the key points, if you want to!)



Assigned Work:

p.51 # 1 - 3