MCR3U

Transformations of Functions
Horizontal Reflection & Scaling

In general, the function $y=f(x)$ can be transformed as $y=af[k(x-p)]+q$.
These transformations are also commonly written as $y=af[b(x-c)]+d$.

The actual letters used are not relevant. It is their meaning that has value. We will use the first form, but it is important to be willing and able to adapt to any form.

Summary (so far):

$y=a(x-h)^2+k$

$$y=af\left[k\left(x-p\right)\right]+q$$

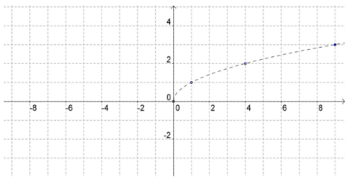
- read transformations from left to right
- if the equation is not in this form, rearrange and/or factor until it matches

Parameter	Value(s)	Effect	Example	
a	$a<0$	vertical reflection	$y=-f(x)$	vertical reflection
	$\|a\|>1$, or $a<-1$ or $a>1$	vertical stretch	$y=4f(x)$	vertical scaling by 4, or vertical stretch by 4
	$\|a\|<1$, or $-1<a<1$	vertical compression	$y=\frac{1}{2}f(x)$	vertical scaling by $\frac{1}{2}$, or vertical compression by 2
p	$p>0$	horizontal shift right by p	$y=f(x-3)$	shift right by 3
	$p<0$	horizontal shift left by p	$y=f(x+5)$	shift left by 5
q	$q>0$	vertical shift up by q	$y=f(x)+7$	shift up by 7
	$q<0$	vertical shift down by q	$y=f(x)-1.5$	shift down by 1.5

Now to consider the final transformations, which are the result of the 'k' parameter. To simplify matters, consider only the k-value in $y=f(kx)$.

Ex.1 Given $f(x)=\sqrt{x}$, complete a table of values for $y=f(-x)$ and graph.

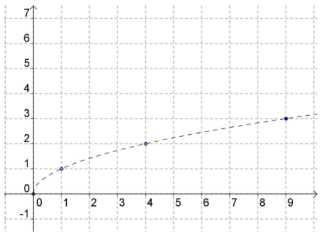
x	$y=\sqrt{x}$	$y=\sqrt{-x}$
-9	inad	
-4	inad	
-1	inad	
0	0	
1	1	
4	2	
9	4	



What is the transformation when $k<0$ (k is negative)?

Ex.2 Given $f(x)=\sqrt{x}$, complete a table of values for $y=f(4x)$ and graph.

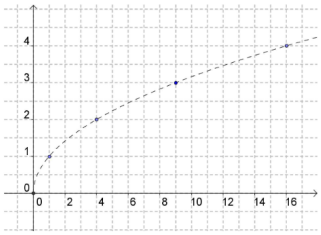
x	$y=\sqrt{x}$	$y=f(4x)$
0	0	0
1	1	
4	2	
9	4	



What is the transformation when $k>1$?

Ex.3 Given $f(x)=\sqrt{x}$, complete a table of values for $y=f\left(\frac{1}{4}x\right)$ and graph.

x	$y=\sqrt{x}$	$y=f\left(\frac{1}{4}x\right)$
0	0	
1	1	
4	2	
16	4	



What is the transformation when $0<k<1$?

To summarize, for $y=f(kx)$

Parameter	Value(s)	Effect	Example	
k	$k<0$	horizontal reflection	$y=f(-x)$	horizontal reflection
	$\|k\|>1$, or $k<-1$ or $k>1$	horizontal compression	$y=f(4x)$	horizontal scaling by $\frac{1}{4}$, or horizontal compression by 4
	$\|k\|<1$, or $-1<k<1$	horizontal stretch	$y=f\left(\frac{1}{2}x\right)$	horizontal scaling by 2, or horizontal stretch by 2

Horizontal Reflection & Scaling

Recall:

Quadratic Relation: (vertex form) $y = a(x-h)^2 + k$ $V(h,k)$

Function Notation: (using a, h, k) $y = af(x-h) + k$

Equivalent Notation: (using a, p, q) $y = af(x-p) + q$

Even if the letters (parameters) change, their meaning remains the same.

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Ex.1 TOV and Graph $y = f(-x)$ given $f(x) = \sqrt{x}$.

x	$\sqrt{x}$	$\sqrt{-x}$
-9	inad	$\sqrt{-(-9)} = \sqrt{9} = 3$
-4	inad	2
-1	inad	1
0	0	0
1	1	inad
4	2	inad
9	3	inad

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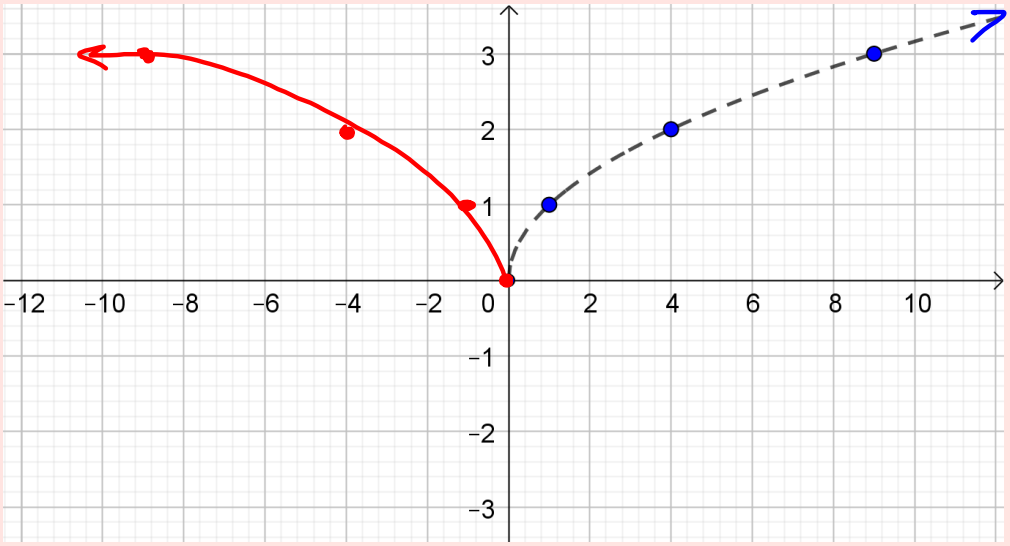
Ex.1 TOV and Graph $y = f(-x)$ given $f(x) = \sqrt{x}$.



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Ex.2 TOV and Graph $y = f(4x)$ given $f(x) = \sqrt{x}$.

x	$\sqrt{x}$	$\sqrt{4x}$
0	0	$\sqrt{4(0)} = 0$
1	1	$\sqrt{4(1)} = 2$
4	2	$\sqrt{4(4)} = 4$
9	3	$\sqrt{4(9)} = 6$

$y = 4f(x)$

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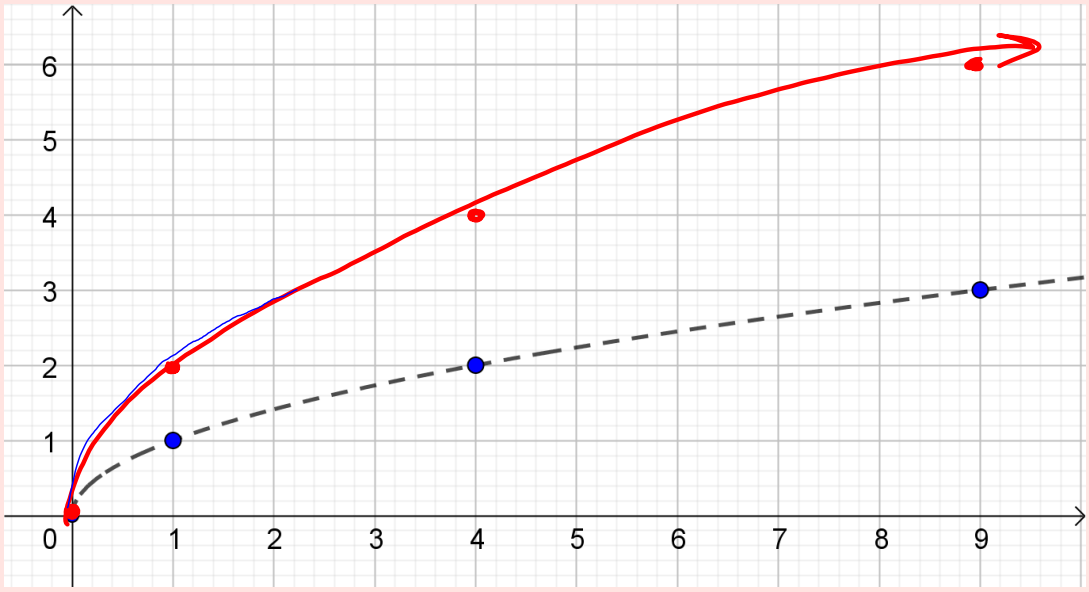
Ex.2 TOV and Graph $y = f(4x)$ given $f(x) = \sqrt{x}$.



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Ex.3 TOV and Graph $y = f(\frac{1}{4}x)$ given $f(x) = \sqrt{x}$.

x	$\sqrt{x}$	$\sqrt{\frac{1}{4}x}$
0	0	0
1	1	$\sqrt{\frac{1}{4}(1)} = \sqrt{\frac{1}{4}} = \frac{1}{2}$
4	2	$\sqrt{\frac{1}{4}(4)} = \sqrt{1} = 1$
16	4	$\sqrt{\frac{1}{4}(16)} = \sqrt{4} = 2$

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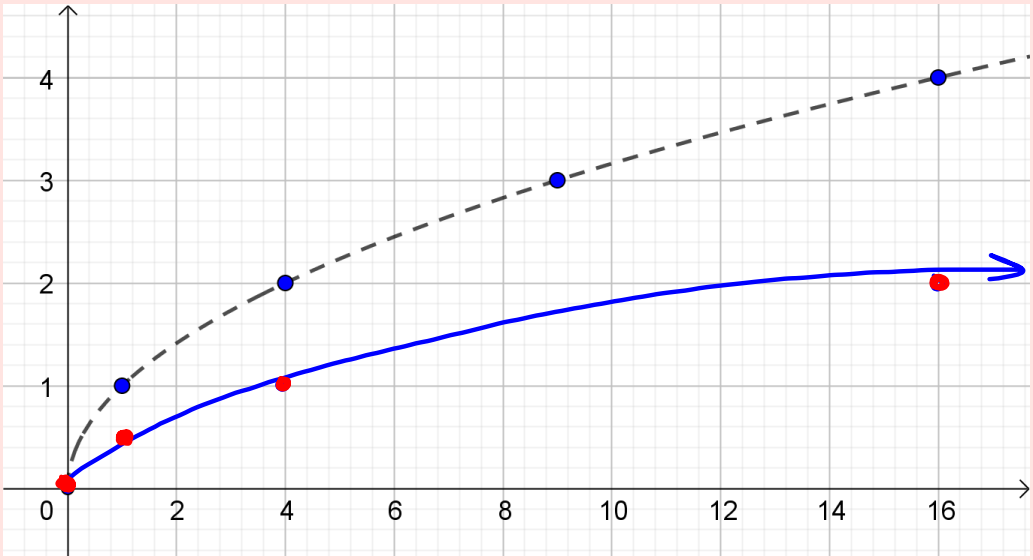
Ex.3 TOV and Graph $y = f\left(\frac{1}{4}x\right)$ given $f(x) = \sqrt{x}$.



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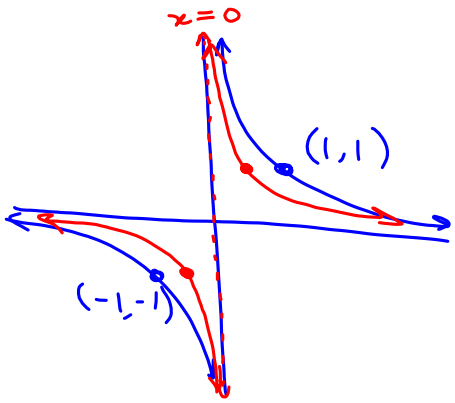
Homework:

p.59 # 3, 4, 7, 8, 11

Note: Functions must be in the form (see 9d)

$$y = af[k(x-p)]+q$$

4(c) $y = \frac{1}{2x}$ $y = \frac{1}{3x}$



$f(x) = \frac{1}{x}$

$f(2x) = \frac{1}{2x}$

↓

h. compress $\frac{x}{2}$ by 2

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