

Rational Functions Graphs

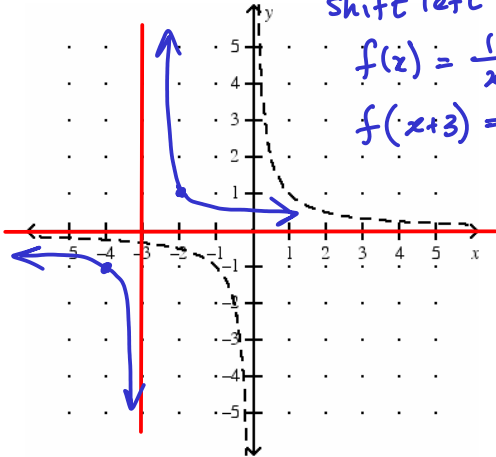
In the functions unit we studied the Reciprocal Function, which is in the family of Rational Functions.

We looked at $y = \frac{1}{x}$

Use your knowledge of functions (and transformations) to sketch the graph of each of the following.

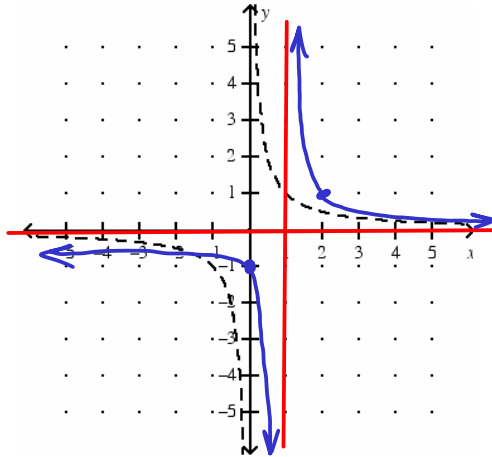
a) $y = \frac{1}{x+3}$

$x \neq -3$
 shift left by 3
 $f(z) = \frac{1}{z}$
 $f(x+3) = \frac{1}{z+3}$



b) $y = \frac{1}{x-1}$

$x \neq 1$
 $f(x) = \frac{1}{x}$
 $f(x-1) = \frac{1}{x-1}$
 shift right 1



Remember: The zeros of the denominator result in vertical asymptotes.

What happens when a factor of the denominator is also a factor of the numerator?

If a factor in the denominator divides out with the same factor in the numerator, the restriction takes the shape of a hole in the graph.

If a factor in the denominator does not divide out, the restriction is a vertical asymptote.

Ex: Simplify the equation of each of the functions, decide whether you have a hole and/or a vertical asymptote, and sketch the graph of the function.

a) $y = \frac{3x-1}{3x^2+5x-2}$

$\begin{matrix} M & -6 \\ A & 5 \\ N & \frac{6}{3}, -\frac{1}{3} \end{matrix}$
 $\frac{3x-1}{(x+2)(3x-1)}$
 $= \frac{1}{x+2}, x \neq \frac{1}{3}$

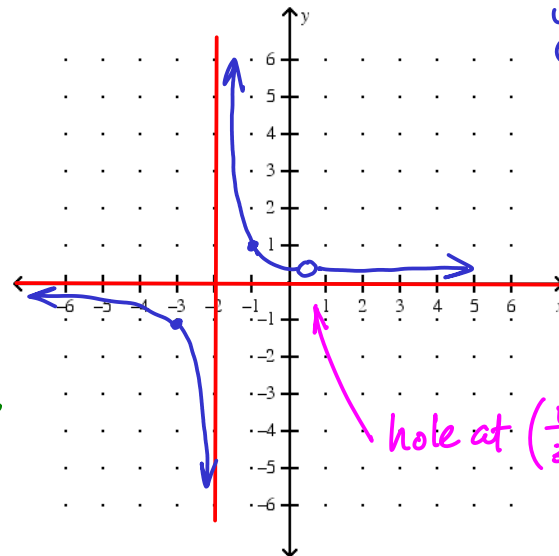
restrict: $x \neq -2, x \neq \frac{1}{3}$

still visible in denominator, vertical asymptote

factored out, hole at $x = \frac{1}{3}$
 $y = \frac{1}{\frac{1}{3}+2}$

$y = \frac{3}{7}$

Sketch:



$y = \frac{1}{x+2}$
 shift $\frac{1}{x}$ to the left by 2

hole at $(\frac{1}{3}, \frac{3}{7})$

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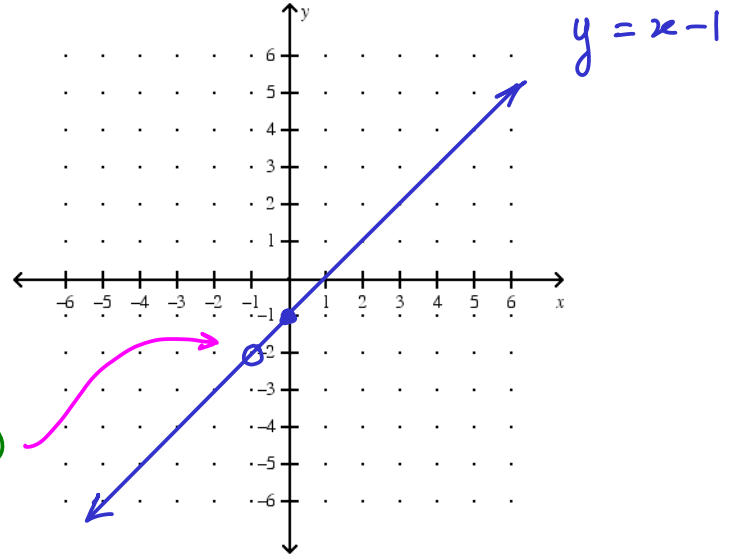
Sketch:

$$b) y = \frac{x^2 - 1}{x + 1}$$

$$= \frac{(x - 1)(x + 1)}{x + 1}$$

$$= x - 1, \quad x \neq -1$$

hole
 $y = (-1) - 1$
 $= -2$ hole at $(-1, -2)$



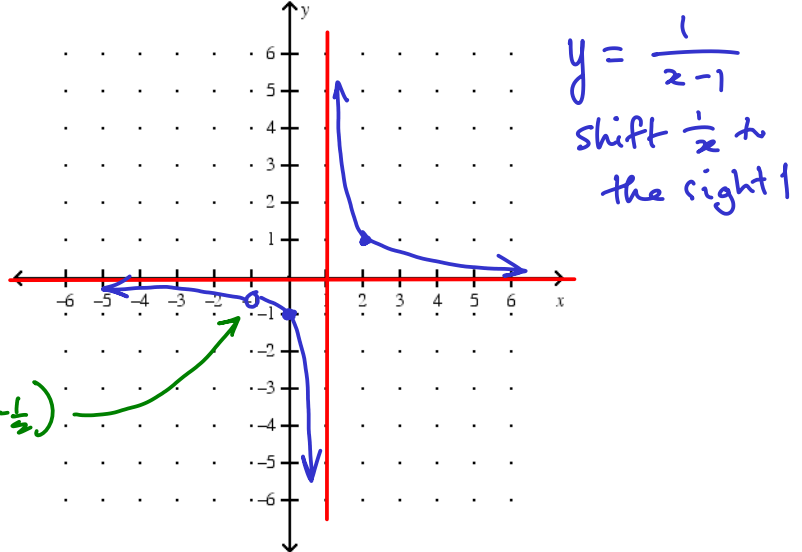
$$c) y = \frac{x + 1}{x^2 - 1}$$

$$= \frac{x + 1}{(x - 1)(x + 1)}$$

$$= \frac{1}{x - 1}, \quad x \neq -1, x \neq 1$$

hole VA: $x = 1$
 $y = \frac{1}{-1 - 1}$
 $= -\frac{1}{2}$ hole: $(-1, -\frac{1}{2})$

Sketch:



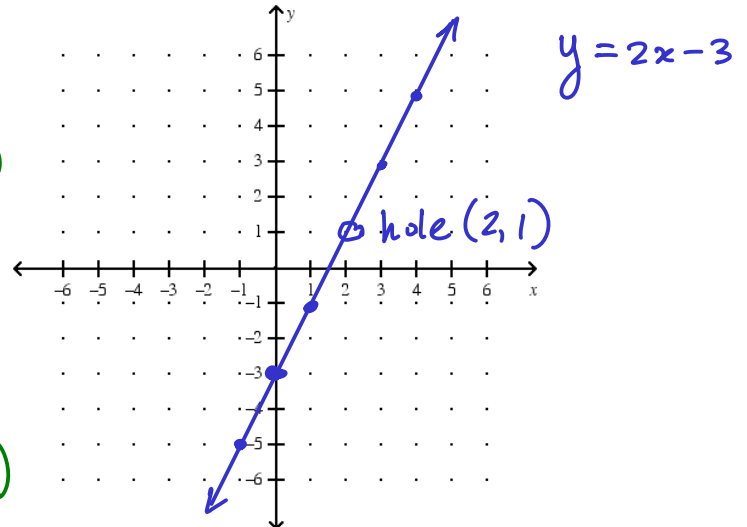
$$d) y = \frac{2x^2 - 7x + 6}{x - 2}$$

M	12
A	-7
N	-3, -4
	$\frac{2}{2} \quad \frac{4}{2}$
	↓ ↓
	$(2x - 3)(x - 2)$

$$= \frac{(2x - 3)(x - 2)}{x - 2}$$

$= 2x - 3, \quad x \neq 2$
 hole
 $y = 2(2) - 3$
 $= 1$ hole @ $(2, 1)$

Sketch:



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e) $y = \frac{x^2 - 9}{4x + 12}$

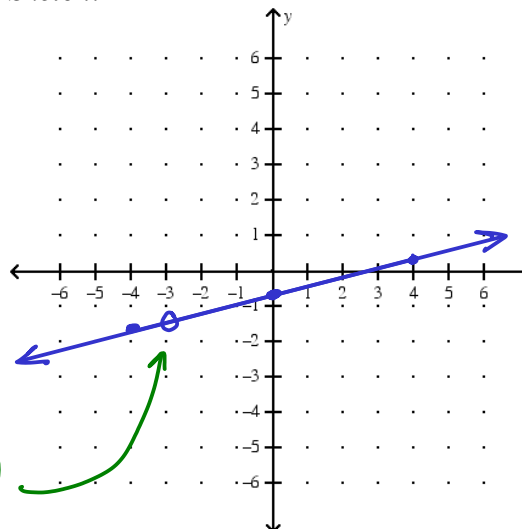
$$= \frac{(x-3)(x+3)}{4(x+3)}$$

$$= \frac{x-3}{4}, \quad x \neq -3$$

hole
↓
 $y = \frac{-3-3}{4}$

$$= -\frac{3}{2} \quad P(-3, -\frac{3}{2})$$

Sketch:



$$y = \frac{x-3}{4}$$
$$= \frac{1}{4}x - \frac{3}{4}$$

* can only
sketch using
this scale



f) $y = \frac{4x - 10}{4x^2 - 25}$

$$= \frac{2(2x-5)}{(2x-5)(2x+5)}$$

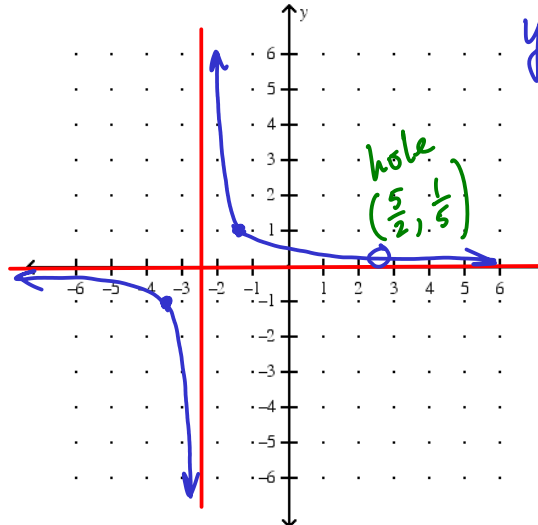
$$= \frac{2}{2x+5}, \quad x \neq \frac{5}{2}, -\frac{5}{2}$$

hole
 $y = \frac{2}{2(\frac{5}{2})+5}$

VA:
 $x = -\frac{5}{2}$

$$= \frac{2}{10}$$
$$= \frac{1}{5} \quad P(\frac{5}{2}, \frac{1}{5})$$

Sketch:



$$y = \frac{2}{2x+5}$$
$$= \frac{2}{2(x+\frac{5}{2})}$$

$$= \frac{1}{x-\frac{5}{2}}$$

shift left $\frac{5}{2}$