Optimal Values Using Symmetry

Oct 8/2019

Our motivation for completing the square is generally to find the vertex of the quadratic relation.

It is also possible to find the vertex from the zeroes, or roots, of the quadratic, and the fact that the parabola is symmetric about the vertex.

In other words, the axis of symmetry is half-way between the zeroes, and it matches the x-coordinate of the vertex.

Sub this value into the equation to find the y-coordinate of the vertex, or the optimal value.

Feb 6-3:52 PM

$y = 35(11-2x)(3x+5)$

Ex. Find the optimal value of $y = -35(2x - 11)(3x + 5)$

① zeroes

$2x - 11 = 0$ $\quad$ $3x + 5 = 0$

$2x = 11$ $\quad$ $3x = -5$

$x = \frac{11}{2}$ $\quad$ $x = -\frac{5}{3}$

② Axis of Symmetry

$$x_v = \frac{\frac{11}{2} + \left(-\frac{5}{3}\right)}{2}$$

$$= \frac{1}{2}\left[\frac{33 - 10}{6}\right]$$

$$= \frac{23}{12}$$

③ Sub x_v to find y_v → optimal value

$$y = -35\left(\cancel{2}^{1}\left(\frac{23}{\cancel{12}_{6}}\right) - \frac{11}{1}\right)\left(3\left(\frac{23}{12}\right) + 5\right)$$

$$= -35\left(\frac{23}{6} - \frac{66}{6}\right)\left(\frac{23}{4} + \frac{20}{4}\right)$$

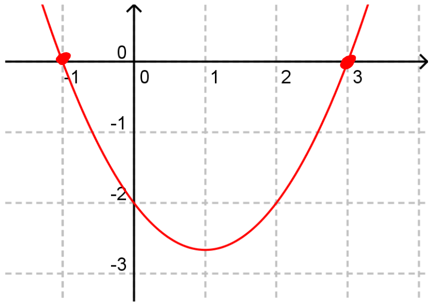
$$= -35\left(-\frac{43}{6}\right)\left(\frac{43}{4}\right)$$

$$y_v = \frac{64715}{24}$$

∴ the optimal value (max) of $\frac{64715}{24}$

Feb 7-9:09 PM

Ex. Given the graph of $y=\frac{2}{3}x^2-\frac{4}{3}x-2$, determine the minimum value.



from graph:

zeroes at $-1, 3$

$$x_v=\frac{-1+3}{2}$$

$$=1$$

$$y=\frac{2}{3}(x+1)(x-3)$$

$$y_v=\frac{2}{3}(1)^2-\frac{4}{3}(1)-2$$

$$=\frac{2}{3}-\frac{4}{3}-\frac{6}{3}$$

$$=\frac{-8}{3}$$

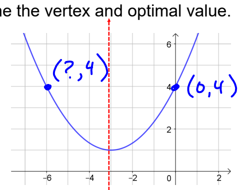
Feb 7-8:57 PM

Ex. Use symmetry to determine the vertex and optimal value.

$$\underbrace{y=\frac{1}{3}x^2+2x+4}_{\text{standard}}$$

y-int at 4

$(0,4)$



do not use graph to solve

by symmetry, there is a matching point with $y=4$.

Set $y=4$: $4=\frac{1}{3}x^2+2x+4$

$$0=\frac{1}{3}x^2+2x$$

$$0=x\left(\frac{1}{3}x+2\right)$$

$x=0$ $\quad$ $\frac{1}{3}x+2=0$

$\frac{1}{3}x=-2$

$x=-6$

Symmetry of matching points

$$x_v=\frac{0+(-6)}{2}$$

$$=-3$$

$$y_v=\frac{1}{3}(-3)^2+2(-3)+4$$

$$=\frac{1}{3}(9)-6+4$$

$=1$ $\quad$ $\therefore$ optimal value of 1.

Oct 8-10:05 AM

Ex.3 The city transit system carries 24,800 bus riders per day for a fare of $3.15. The city hopes to reduce car pollution by getting more people to ride the bus, while maximizing the transit system's revenue at the same time. A survey indicates that the number of riders will increase by 800 for every $0.05 decrease in the fare. What fare will produce the greatest revenue?

$R = (\text{price})(\#\text{customers})$

$R_1 = (3.15)(24800)$

$R_2 = (3.15 - 0.05)(24800 + 800)$

$R_3 = (3.15 - 2(0.05))(24800 + 2(800))$

$R(x) = (3.15 - 0.05x)(24800 + 800x)$

Let x represent the number of \#0.05 decreases.

Feb 7-9:20 PM

Assigned Work:

Solve by factoring or symmetry
p.153 # 4acd, 5, 8, 12, 15 + worksheet 2, 5, 1

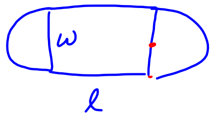
Note: #15 in "extending" section is acommon question

5. $R(x) = x \cdot p(x)$

$R(x) = x(-x + 5)$

Feb 1-7:30 PM

12.



$C = 2\pi r$

$= \pi d$

$= \pi w$

$P = 500$

$500 = 2\ell + C$

$500 = 2\ell + \pi w$

$500 - \pi w = 2\ell$

$\ell = \frac{500 - \pi w}{2}$

$A = \ell w$

$A(w) = \left(\frac{500 - \pi w}{2}\right)(w)$

$\downarrow$

max ?

① zeroes

② x_{vertex} (A of S)

③ $y_v \rightarrow$ don't care

① zeroes of $A(w)$

$\frac{500 - \pi w}{2} = 0$

$500 - \pi w = 0$

$500 = \pi w$

$w = \frac{500}{\pi}$ or $w = 0$

$w_v = \frac{\frac{500}{\pi} + 0}{2}$ $\qquad \frac{1}{2} \cdot \frac{500}{\pi}$

$= \frac{250}{\pi}$

Oct 9-2:05 PM

15. $R = (8)(300)$

$R = (8+1)(300-30)$

$R = (8+2(1))(300-2(30))$

$R(x) = (8+x)(300-30x)$

where x is the number of \$1 price increases.

① zeroes: -8 and 10

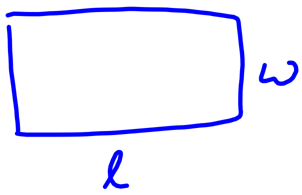
② $x_v = \frac{-8+10}{2}$

$= 1$

$\therefore$ new ticket price is \$9.

Oct 9-2:13 PM

WS # 1.



$P = 500$

$2(l+w) = 500$

$l + w = 250$

$l = 250 - w$

$A = lw$

$A(w) = (250-w)(w)$

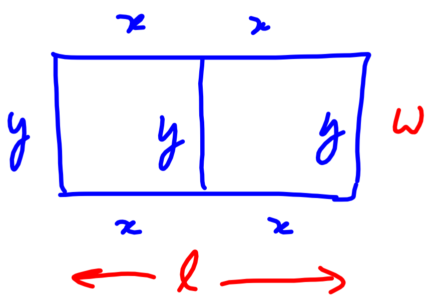
① zeroes

② x_{vertex}

③ y_{vertex}.

Oct 9-2:16 PM

2.



$1200 = 4x + 3y$

$A = 2xy$

✓ $1200 = 2l + 3w$

$600 = l + w$

$A = lw$

Oct 9-2:18 PM

#sold price

5. $R(x) = (90 + 5x)(200 - 10x)$

where x is the number of \$10 price decreases.

① zeroes
② $x_v = 1$
③

(b) Set $R(x) = 15600$

$15600 = \underbrace{(90+5x)(200-10x)}_{\text{expand}}$

⋮

$0 = [\qquad\qquad] - 15600$

$0 = \underbrace{ax^2 + bx + c}_{\text{GQF}}$

Oct 9-2:22 PM