Unit 4 - Exponential Functions

Oct 23/2019

Review of Exponent Laws

Assigned Work: (1, 2, 4-9, 11, 13, 16, 18)(odd letters)

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Recall:

A **power** is a product of identical factors and consists of two parts: a **base** and an **exponent**.

$$2^3 = (2)(2)(2)$$

base = the identical factor

exponent = how many factors there are altogether

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Ex. Evaluate.

a) 3^3

$= (3)(3)(3)$

$= 27$

b) $(-3)^3 = (-3)(-3)(-3)$

$= -27$

c) -2^4 BEDMAS

$= -1(2)^4$

$= -(2)^4$

$= -(16)$

$= -16$

d) $\left(\frac{4}{3}\right)^2 = \left(\frac{4}{3}\right)\left(\frac{4}{3}\right)$

$= \frac{16}{9}$

$= \frac{4^2}{3^2}$

$-1x = -1(x)$

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Rule #1: Multiplication of Powers with the same base

To investigate the rule let us look at a specific example and go through the process of expanding before simplifying.

$(3^1)(3^2) = (3)(3)(3)$

$= 3^3$

$= 3^{1+2}$

The Rule: $(a^x)(a^y) = a^{x+y}$

In words: when multiplying powers with the same base, add exponents.

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Rule #2: Division of Powers with the same base

To investigate the rule let us look at a specific example and go through the process of expanding before simplifying.

$3^1 \div 3^2 = \frac{3^1}{1(3)(3)}$

$= \frac{1}{3}$

$= 3^{-1}$

The Rule: $a^x \div a^y = \frac{a^x}{a^y}$

$= a^{x-y}, \ a \neq 0$

In words: when dividing powers with the same base, subtract exponents.

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Rule #3: Power of a Power

To investigate the rule let us look at a specific example and go through the process of expanding before simplifying.

$\left(3^2\right)^4 = (3^2)(3^2)(3^2)(3^2)$

$= 3^{2+2+2+2}$

$= 3^8$

The Rule: $\left(a^x\right)^y = a^{xy}$

In words: when having a power to an exponent, multiply the exponents.

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Ex.1 Simplify. Express your final answer with a single base and exponent.

(a) $\left(4^{-6}\right)\left(4^4\right)$

$= 4^{-6+4}$

$= 4^{-2} = \frac{1}{16}$

(b) $\dfrac{(-3)^2}{(-3)^{-3}} = (-3)^{2-(-3)}$

$= (-3)^5$

(c) $\left(5^{-2} \times 5^4\right)^{-2}$

$= \left(5^2\right)^{-2}$

$= 5^{-4}$

(d) $\left(3a^2b\right)^1\left(-2a^3b^4\right)^1$

$= -6a^5b^5$

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Rule #4: Identity Rule

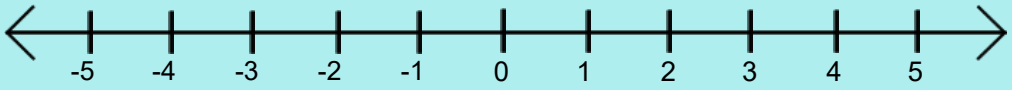
What exponent does not change the value of a power?

The Rule: $a^1 = a$

In words: anything to the exponent of 1 is equal to itself.

Patterns in Powers of 2

exponent



2^0 2^1 2^2 2^3

1 2 4 8

×2 ×2

÷2 ÷2 ÷2

5^0 5^1 5^2 5^3

1 5 25 125

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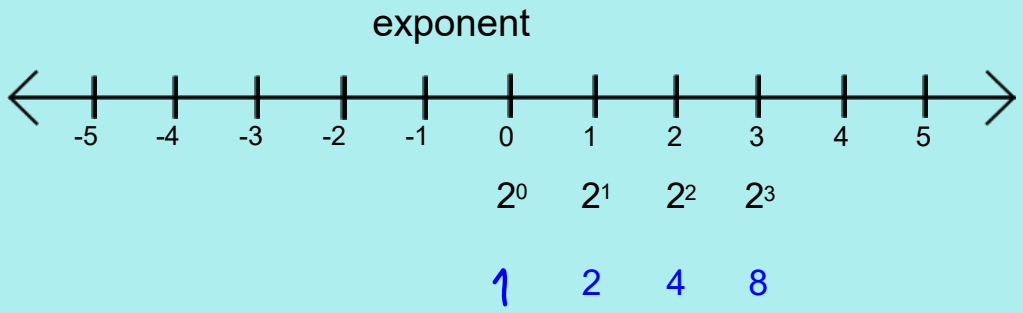
Patterns in Powers of 2



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Rule #5: Zero Exponent

Lets look at the expanded form of powers and find a pattern:

$2^3 =$ 8 ÷2

$2^2 =$ 4 ÷2

$2^1 =$ 2 ÷2

then $2^0 =$ 1

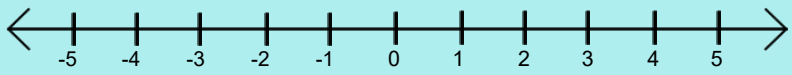
The Rule: $a^0 = 1,\ a \neq 0$

In words: anything to the exponent of zero is 1.
This is because an exponent of zero means you are dividing the base by itself.

0^0 is undefined

Patterns in Powers of 2

exponent



2^{-3} 2^{-2} 2^{-1} 2^{0} 2^{1} 2^{2} 2^{3}

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{2}$ 1 2 4 8

$\frac{1}{2^3}$ $\frac{1}{2^2}$ $\frac{1}{2^1}$ $\div 2$

$5^{-2} = \frac{1}{5^{-2}}$ $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2}$

$\frac{2^3}{5^{-2}} = \frac{2^3}{1}\left(\frac{1}{5^{-2}}\right)$

$= 2^3(5^2)$

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Rule #6: Negative Exponent

Now continue the pattern from the previous rule to determine the effect of a negative in the exponent:

$2^0 =$ 1

$2^{-1} =$ $\frac{1}{2} = \frac{1}{2^1}$

$2^{-2} =$ $\frac{1}{4} = \frac{1}{2^2}$

The Rule: $a^{-x} = \left(\frac{1}{a}\right)^{x}, \ a \neq 0$

In words: a negative exponent requires you to find the reciprocal of the base.

Rule #7: Distributive Rule (for powers with different bases)

To investigate the rule let us look at a specific example and go through the process of expanding before simplifying.

$\left(7^2 \cdot 2^5\right)^3 = (7^2 \cdot 2^5)(7^2 \cdot 2^5)(7^2 \cdot 2^5)$

$= 7^6 \cdot 2^{15}$

$\left(\frac{7^2}{2^5}\right)^3 = \frac{7^6}{2^{15}}$

The Rules:

(a) $(ab)^x = (a^x)(b^x)$

(b) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x},\ b \neq 0$

(c) $\left(a^m b^n\right)^p = a^{mp} b^{np}$

(d) $\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}$

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Ex.2 Simplify. Express your final answer with a single base and positive exponent.

(a) $\left(4^{-6}\right)\left(4^4\right)$

$= 4^{-2}$

$= \frac{1}{4^2}$

(b) $\left(5^{-2} \times 5^4\right)^{-2}$

$= (5^2)^{-2}$

$= 5^{-4}$

$= \frac{1}{5^4}$

(c) $\left(x^{-3} y^5\right)^{-3}$

$= x^9 y^{-15}$

$= \frac{x^9}{y^{15}}$

(d) $\left(-a^5 b\right)^{-2}\left(-ab^{-2}\right)^2$

$= [(-1)^{-2} a^{-10} b^{-2}][(-1)^2 a^2 b^{-4}]$

$= (-1)^0 a^{-8} b^{-6}$

$= \frac{1}{a^8 b^6}$

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The exponent laws also work if you have polynomials instead of numbers as exponents.

Ex.3 Simplify

(a) $\left(x^3\right)^{2a+4}$

$= x^{3(2a+4)}$

$= x^{6a+12}$

(b) $\left(x^{a+5}\right)\left(x^{3a+1}\right)$

$= x^{(a+5)+(3a+1)}$

$= x^{4a+6}$

(c) $\left(x^{4m-3n}\right) \div \left(x^{m+5n}\right)$

$= x^{(4m-3n)-(m+5n)}$

$= x^{3m-8n}$

(d) $x^y \left(x^{y+1}\right)^{y+2} \left(\frac{1}{x}\right)^{6y}$

$= x^y \, x^{(y+1)(y+2)} \, x^{-6y}$

$= x^y \, x^{y^2+3y+2} \, x^{-6y}$

$= x^{y^2-2y+2}$

$\left(\frac{1}{x}\right)^{6y} = \frac{1^{6y}}{x^{6y}}$

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Summary - The exponent laws

same base

$$(a^x)(a^y) = a^{x+y}$$

$$a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}, \quad a \neq 0$$

$$a^{-x} = \frac{1}{a^x}, \quad a \neq 0$$

$$\left(a^x\right)^y = a^{xy}$$

$$a^0 = 1, \quad a \neq 0$$

different bases

$$(ab)^x = \left(a^x\right)\left(b^x\right)$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}, \quad b \neq 0$$

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Assigned Work:

(1, 2, 4-9, 11, 13, 16, 18)(odd letters)
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7(a) $16^{-1} - 2^{-2}$

$= \frac{1}{16} - \frac{1}{2^2}$

$= \frac{1}{16} - \frac{1}{4}$

$= \frac{1}{16} - \frac{4}{16}$

$= \frac{-3}{16}$

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13(i) $\frac{(5^{-1}) - (2^{-2})}{(5^{-1}) + (2^{-2})}$

$= \frac{\left(\frac{1}{5}\right) - \frac{1}{4}}{\frac{1}{5} + \frac{1}{4}}$

$= \frac{\frac{4-5}{20}}{\frac{4+5}{20}}$

$= \frac{-1}{20} \times \frac{20}{9}$

$= \frac{-1}{9}$

$\frac{(1/5 - 2)}{(1/5 + 2)}$

$\frac{(5^{-1}) - 2^{-2}}{5^{-1} + 2^{-2}}$

$= (0.2)$

$\frac{3^{-2}}{4^{-5}} = \frac{4^5}{3^2}$

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13(e) $\dfrac{2^5}{3^{-2}} \cdot \dfrac{3^{-1}}{2^4}$

$= \dfrac{(2^5)(3^{-1})}{(3^{-2})(2^4)}$

$\dfrac{ab}{cd} = \dfrac{ba}{dc}$

$= (2^{5-4})(3^{-1-(-2)})$

$= 2(3)$

$= 6$

$= \dfrac{(2^5)(3^2)}{(2^4)(3^1)}$

$= \dfrac{(32)(9)}{(16)(3)}$

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18(e) $(a^{10-p})\left(\frac{1}{a}\right)^p \longrightarrow (a^{10-p})\left(\frac{1^p}{a^p}\right)$

$= (a^{10-p})(a^{-p})$

$= a^{10-2p}$

16. (e) $25^n = \frac{1}{625} \rightarrow \frac{1}{25^{-n}} = \frac{1}{25^2}$

$25^n = \frac{1}{25^2} \Rightarrow -n = 2$

$25^n = 25^{-2} \quad n = -2$

$\Rightarrow$ "implies" $n = -2$

$25^n = \frac{1}{3125}$

$(5^2)^n = \frac{1}{5^5}$

$5^{2n} = 5^{-5}$

$\Rightarrow 2n = -5$

$n = \frac{-5}{2}$

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