Determining Exponential Equations (from Graphs/Points)

Oct 30/2019

Only vertical transformations are needed. All horizontal transformation can be removed by using an equivalent form.

horizontal transformation		equivalent form	
$y = 2^{-x}$	horizontal reflection	$y = \left(\frac{1}{2}\right)^x$	change base
$y = 2^{x+1}$ $=(2^x)(2^1)$	shift left by 1	$y = 2(2^x)$	vertical stretch by 2
$y = 2^{3x}$	horizontal compression by 3	$\left(2^3\right)^x = 8^x$	change base

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Target equation: $y = a\left(b^x\right) + q$ (vertical only)

(1) The 'q' value represents a vertical shift, and is the only transformation to affect the horizontal asymptote.

$$y = 0 \longrightarrow y = q$$

(2) The 'b' value is the common ratio for the exponential function.

(a) With no vertical shift (q=0), use ratios of y-coordinates.

$$y_2 \div y_1$$

(b) If there is a vertical shift, use ratios of distances from HA to each known point.

$$d_2 \div d_1$$

(c) In general, take the ratio of first differences.

$$\Delta y_2 \div \Delta y_1$$

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Note: 'b' calculations assume consecutive x-values: $\Delta x = 1$
Otherwise, adjust your equation using horizontal scaling.

$$\Delta x \qquad y = a\left(b^{\frac{x}{\Delta x}}\right) + q$$

(3) Both 'q' and 'a' affect the y-intercept (and all other y-values). Once 'q' is known, sub the new y-int to find 'a', or use the mapping definition.

$$(0,1) \longrightarrow (0, a+q)$$

Note: This assumes p=0

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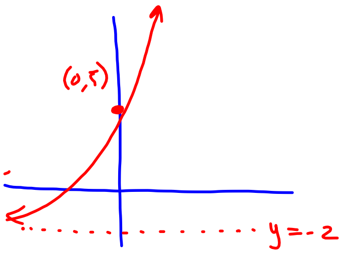
Ex. Determine the equation of the exponential function with a common ratio of 3, a y-intercept of 5 and a horizontal asymptote at y=-2.

$y = ab^x + q$



$b = 3$ (common ratio)

HA: $y=0 \rightarrow y=-2 \qquad q=-2$

$$y = a(3^x) - 2$$

sub point (0,5)

$5 = a(3^0) - 2$

$7 = a(1)$

$a = 7$

use y-int transforms:

$(0,1) \rightarrow (0, a+q)$

$\rightarrow (0,5)$

$a+q = 5$

$a-2 = 5$

$a = 7$

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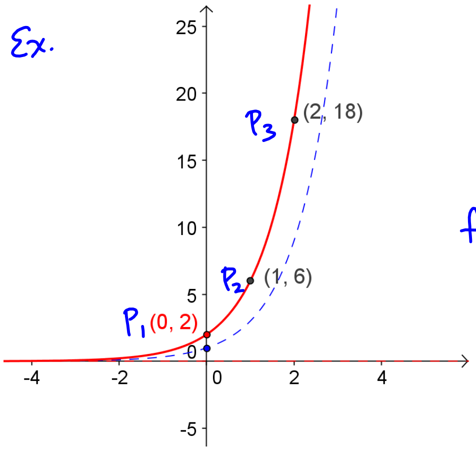
Ex.



HA: $y = 0$

$\boxed{q = 0}$

for b (common ratio)

$\frac{y_2}{y_1} = \frac{6}{2} = 3$

$\frac{y_3}{y_2} = \frac{18}{6} = 3$

$\boxed{b = 3}$

$y = a(3^x) + 0$

$y = a(3^x)$

sub $(0,2)$ or $(1,6)$ or $(2,18)$

$2 = a(3^0)$

$\boxed{2 = a}$

$y = 2(3^x)$

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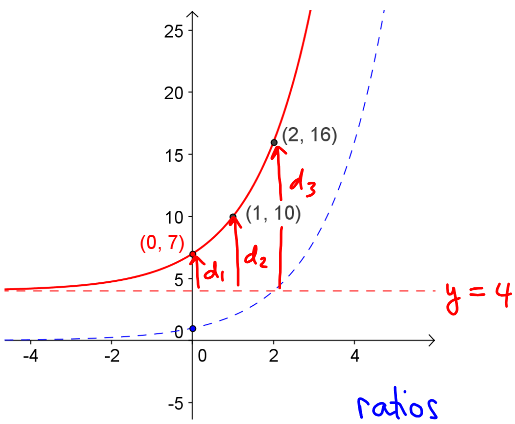

HA: $y = 4$, $\boxed{q = 4}$

ratios

$\frac{d_2}{d_1} = \frac{6}{3} = 2$ $\frac{d_3}{d_2} = \frac{12}{6} = 2$

$\boxed{b = 2}$

$y = a(2^x) + 4$

sub $(0,7)$

$7 = a(2^0) + 4$

$\boxed{3 = a}$

$\boxed{y = 3(2^x) + 4}$

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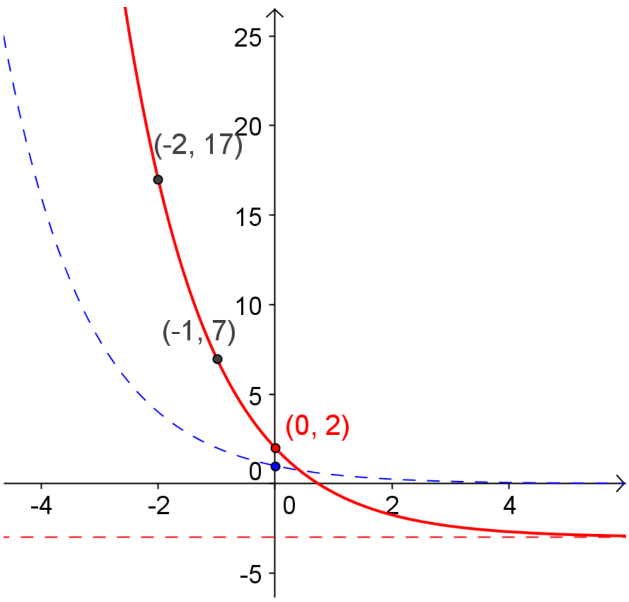

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Assigned Work:

worksheet

$y = ab^x + q$

3. $\left.\begin{array}{l} b = 2 \\ q = 6 \end{array}\right\}$ $y = a\cdot(2^x) + 6$

sub $(0, -3)$

$-3 = a(\cancel{2^0}) + 6$ 1

$-9 = a$

$\boxed{y = -9(2^x) + 6}$

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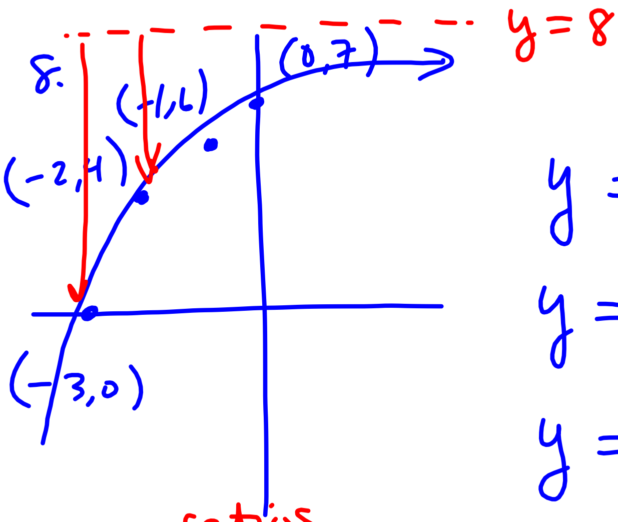

ratios

$d_1 = 8$
$d_2 = 4$ — $\frac{1}{2}$
$d_3 = 2$ — $\frac{1}{2}$
$d_4 = 1$ — $\frac{1}{2}$

$$y = a(b^x) + q$$
$$y = a(b^x) + 8$$
$$y = a\left(\tfrac{1}{2}\right)^x + 8$$

Sub $(0,7)$

$$7 = a\left(\tfrac{1}{2}\right)^0 + 8$$
$$-1 = a$$

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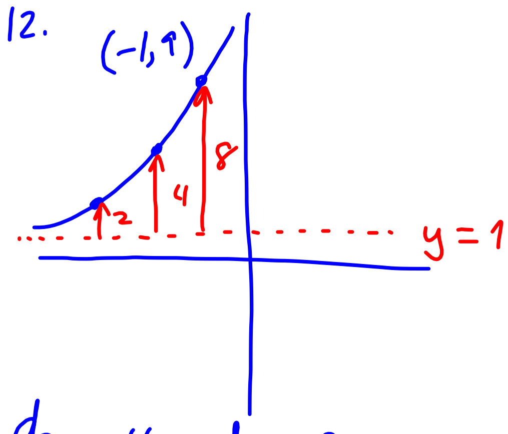

$$\frac{d_2}{d_1} = \frac{4}{2} = 2 \qquad \frac{d_3}{d_2} = \frac{8}{4} = 2$$

$$y = a(b^x) + 1$$
$$y = a(2^x) + 1$$

sub $(-1,9)$

$$9 = a(2^{-1}) + 1$$
$$8 = a\left(\tfrac{1}{2}\right)$$
$$16 = a$$

$$\boxed{y = 16(2^x) + 1}$$

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14. $y = 4(32)^{-\frac{1}{5}(x-1)} + 4$

$= 4\left[32^{-\frac{1}{5}}\right]^{(x-1)} + 4$

$= 4\left[\frac{1}{2}\right]^{x-1} + 4$

$= 4\left[\frac{1}{2}\right]^{x}\left[\frac{1}{2}\right]^{-1} + 4$

$= 4\left(\frac{1}{2}\right)^{x}\left(\frac{2}{1}\right) + 4$

$\boxed{y = 8\left(\frac{1}{2}\right)^{x} + 4}$

$32^{-\frac{1}{5}}$

$= \frac{1}{32^{\frac{1}{5}}}$

$= \frac{1}{\sqrt[5]{32}}$

$= \frac{1}{2}$

$4(x)(2)$

$= 4(2)(x)$

$= 8x$

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15. $y = -2(9)^{-\frac{1}{2}(x+4)} - 1$

$= -2\left[(9)^{-\frac{1}{2}}\right]^{x+4} - 1$

$= -2\left(\frac{1}{3}\right)^{x+4} - 1$

$= -2\left(\frac{1}{3}\right)^{x}\left(\frac{1}{3}\right)^{4} - 1$

$= -2\left(\frac{1}{3}\right)^{x}\left(\frac{1}{81}\right) - 1$

$= \frac{-2}{81}\left(\frac{1}{3}\right)^{x} - 1$

$9^{-\frac{1}{2}}$

$= \frac{1}{9^{\frac{1}{2}}}$

$= \frac{1}{\sqrt[2]{9}}$

$= \frac{1}{3}$

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17.

x	y	Δy
-5	$\frac{77}{3}$	
		$\frac{10}{3}$
-4	29	
		20
-3	49	
		120
-2	169	

$29 - \frac{77}{3}$
$= \frac{87}{3} - \frac{77}{3}$
$= \frac{10}{3}$

common ratio: $\frac{\Delta y_2}{\Delta y_1} = \frac{20}{\frac{10}{3}} = \frac{20}{1} \cdot \frac{3}{10} = 6$ $\quad \frac{\Delta y_3}{\Delta y_2} = \frac{120}{20} = 6$

$y = a(6^x) + q$ Sub 2 points, solve system.

Sub $(-2, 169)$ $\quad (-3, 49)$

$169 = a(6^{-2}) + q \quad 49 = a(6^{-3}) + q$

$169 = \frac{1}{36}a + q$ ① $\quad 49 = \frac{1}{216}a + q$ ②

$49 = \frac{1}{216}a + q$

$120 = \frac{6 \times 1}{6 \times 36}a - \frac{1}{216}a$

$120 = \frac{6a - a}{216}$

$\cancel{120}^{24}(216) = \frac{\cancel{5}a}{\cancel{5}}$

$a = 5184$

$y = 5184(6^x) + q$

Sub $(-2, 169)$

$169 = 5184(6^{-2}) + q$

$169 = 5184\left(\frac{1}{36}\right) + q$

$169 = 144 + q$

$25 = q$

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# bounces	height$_1$	ratios	h_2
0	129.06		98.72
		0.77	
1	100.02		41.61
		0.77	
2	77.51		17.54
		0.79	
3	61.26		7.58
		0.79	
4	48.46		3.36
		0.79	
5	38.69		1.51

$\bar{r} = 0.78$

$q = 0$

$h(n) = a(0.78)^n$

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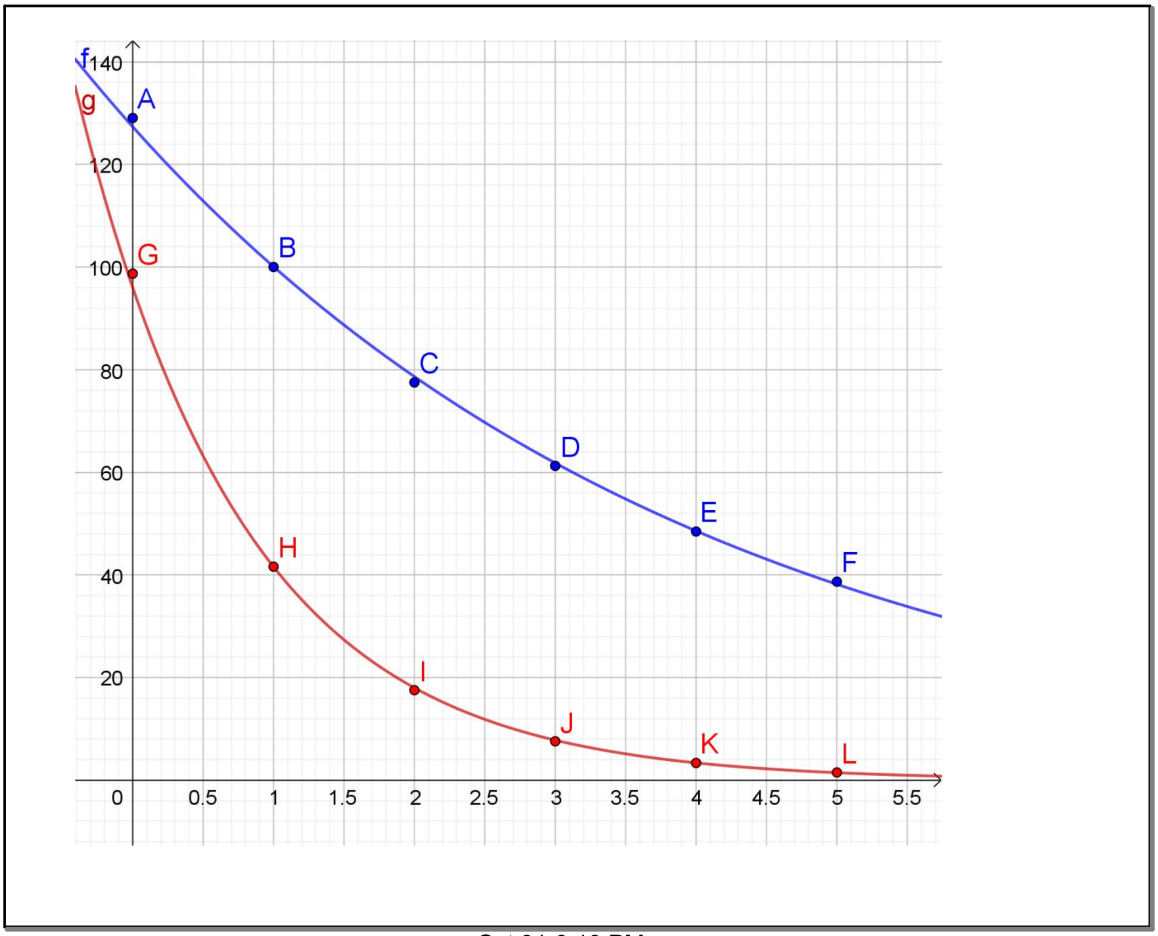

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18.

Δx	x	y	Δy
	-11	$-\frac{31}{6}$	
$+4$	-7	$-\frac{67}{6}$	$\frac{36}{6}$
$+4$	-3	$-\frac{79}{6}$	$\frac{12}{6}$
$+4$	1	$-\frac{83}{6}$	$\frac{4}{6}$

ratios

$$\frac{12}{6} \div \frac{36}{6} = \frac{12}{6} \cdot \frac{6}{36} = \frac{1}{3}$$

$$\frac{4}{6} \div \frac{12}{6} = \frac{4}{6} \cdot \frac{6}{12} = \frac{1}{3}$$

$$b = \frac{1}{3}$$

$$y = a b^{\frac{x}{\Delta x}} + q$$

$$y = a b^{\frac{x}{4}} + q$$

sub $\left(1, -\frac{83}{6}\right)$ $\quad \left(-3, -\frac{79}{6}\right)$

$$-\frac{83}{6} = a\left(\frac{1}{3}\right)^{\frac{1}{4}} + q$$

$$-\frac{79}{6} = a\left(\frac{1}{3}\right)^{-\frac{3}{4}} + q$$

$$-\frac{4}{6} = a\sqrt[4]{\tfrac{1}{3}} - a\left(\left(\sqrt[4]{3}\right)^3\right)$$

$ax + by + c = 0$

$dx + ey + f = 0$

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