Solving Exponential Equations

Nov. 1/2019

Exercises: Handout (photocopy) # (1 - 11, 19, 20) (odd)

To determine the exact solution for an exponential equation, all powers should have the same base. Even then, an exact solution may not always be possible.

We can also solve using various techniques employed for solving equations in other contexts, such as:

- linear systems (substitution, elimination)
- quadratic relations (factoring, solve for zeroes)
- guess & check (gives approximate answers)

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Examples. Solve:

1. (get a common base first)

(a) $2^x = 16$ (b) $5^{2x-1} = \frac{1}{125}$

2. (common base, order of operations)

(a) $8^{x+4} = 4^5$ (b) $3\left(4^x\right) = 48$

3. (guess & check)

(a) $2^x = 9$ (b) $3^{2x-5} = 25$

4. (base, quadratic equation, let 'a' represent...)

(a) $2^{x^2+2x} = \frac{1}{2}$ (b) $2^{2x} - 2^x = 12$

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Examples. Solve:

1. (get a common base first)

(a) $2^x = 16$

$2^x = 2^4$

$\Rightarrow x = 4$

"implies"

(b) $5^{2x-1} = \frac{1}{125}$

$5^{2x-1} = 125^{-1}$

$5^{2x-1} = (5^3)^{-1}$

$5^{2x-1} = 5^{-3}$

$\Rightarrow 2x-1 = -3$

$2x = -2$

$x = -1$

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2. (common base, order of operations)

(a) $8^{x+4} = 4^5$

$(2^3)^{x+4} = (2^2)^5$

$2^{3x+12} = 2^{10}$

$\Rightarrow 3x+12 = 10$

$3x = -2$

$x = -\frac{2}{3}$

(b) $\frac{3(4^x)}{3} = \frac{48}{3}$

$4^x = 16$

$4^x = 4^2$

$\Rightarrow x = 2$

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3. (guess & check)

(a) $2^x = 9$

between 3, 4: $2^3 = 8$, $2^4 = 16$ } 9 is between

try $x = 3.2$

$2^{3.2} \doteq 9.20$

$2^{3.1} \doteq 8.6$

$2^{3.15} \doteq 8.9$

$2^{3.16} \doteq 8.94$

$2^{3.17} \doteq 9.00046$

(b) $3^{2x-5} = 25$

let $a = 2x - 5$

$3^a = 25$

$3^2 = 9$

$3^3 = 27$

$3^{2.9} \doteq 24.19$

$3^{2.95} \doteq 25.55$

$3^{2.93} \doteq 25.001$

$2x - 5 \doteq 2.93$

$2x \doteq 7.93$

$x \doteq 3.965$

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4. (base, quadratic equation, let 'a' represent...)

(a) $2^{x^2+2x} = \frac{1}{2}$

$2^{x^2+2x} = 2^{-1}$

$\Rightarrow x^2 + 2x = -1$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$\boxed{x = -1}$

(b) $2^{2x} - 2^x = 12$

$(2^x)^2 - 2^x = 12$

let $a = 2^x$

$a^2 - a = 12$

$a^2 - a - 12 = 0$

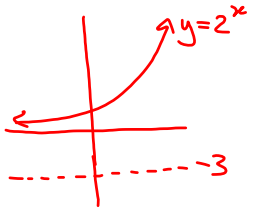
$(a-4)(a+3) = 0$

$a = 4$ $\quad$ $a = -3$

$2^x = 4$ $\quad$ $2^x = -3$ (no solution)

$2^x = 2^2$

$\Rightarrow \boxed{x = 2}$



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Assigned Work:

Handout (photocopy) # (1 - 11, 19, 20) (odd)

and...

(a) $3^{x+2} - 3^x = 216$

(b) $25^x - 30\left(5^x\right) + 125 = 0$

10e c
4a
11e c
19a
20c

11(b) $4^{x+3} + 4^x = 260$

$(4^x)(4^3) + 4^x = 260$

$64(4^x) + 4^x = 260$

$65(4^x) = 260$

$4^x = 4^1$

$\Rightarrow \boxed{x = 1}$

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4(a) $16^{2x} = 8^{3x}$

$(2^4)^{2x} = (2^3)^{3x}$

$2^{8x} = 2^{9x}$

$\Rightarrow 8x = 9x$

$0 = 9x - 8x$

$0 = x$

10(c) $5(4^x) = 10$

$4^x = 2$

$2^{2x} = 2^1$

$\Rightarrow 2x = 1$

$x = \frac{1}{2}$

10(e) $\frac{2}{6} = \frac{6\left(3^{4f-2}\right)}{6}$

$\frac{1}{3} = 3^{4f-2}$

$3^{-1} = 3^{4f-2}$

$\Rightarrow -1 = 4f - 2$

$1 = 4f$

$f = \frac{1}{4}$

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11 ce

(c) $2^{a+5} + 2^a = 1056$

$(2^a)(2^5) + 2^a = 1056$

$32(2^a) + 1(2^a) = 1056$

$33(2^a) = 1056$

$2^a = 32$

$2^a = 2^5$

$\Rightarrow a = 5$

(e) $3^{x+3} - 3^{x+1} = 648$

$(3^x)(3^3) - 3^x(3^1) = 648$

$27(3^x) - 3(3^x) = 648$

$24(3^x) = 648$

$3^x = 27$

$3^x = 3^3$

$\Rightarrow x = 3$

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19a 20c

19(a) $\dfrac{27^x}{9^{2x-1}} = 3^{x+4}$

$\dfrac{(3^3)^x}{(3^2)^{2x-1}} = 3^{x+4}$

$\dfrac{3^{3x}}{3^{(4x-2)}} = 3^{x+4}$

$3^{3x-(4x-2)} = 3^{x+4}$

$\Rightarrow -x + 2 = x + 4$

$-2 = 2x$

$\boxed{x = -1}$

$\dfrac{3^x}{3^y} = 3^{x-y}$

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20(c) $2^{2x^2-3x} = 2^{x^2-2x+12}$

$\Rightarrow 2x^2-3x = x^2-2x+12$

$x^2-x-12=0$

$(x-4)(x+3)=0$

$x=4$ ✓ $x=-3$ ✓

for a^x, any value of x is acceptable

$x \in \mathbb{R}$

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1(c) $(-1)^m = -1$

exponent needs to be <u>odd</u> to give negative result.

eg. $m=-\frac{1}{3}$

$(-1)^{-\frac{1}{3}} = \frac{1}{(-1)^{\frac{1}{3}}}$

$= \frac{1}{\sqrt[3]{-1}}$

$= \frac{1}{-1}$

$= -1$

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