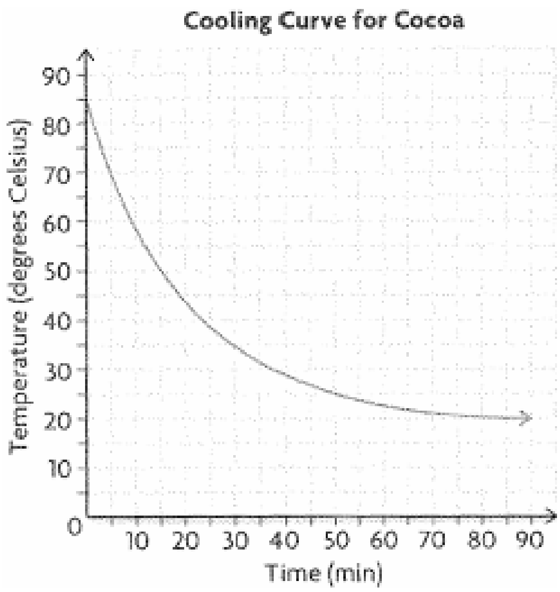
Exponential Applications (part 1)

Nov. 4/2019

Ex.1 A cup of hot cocoa left on a desk in a classroom had its temperature measured once every minute...



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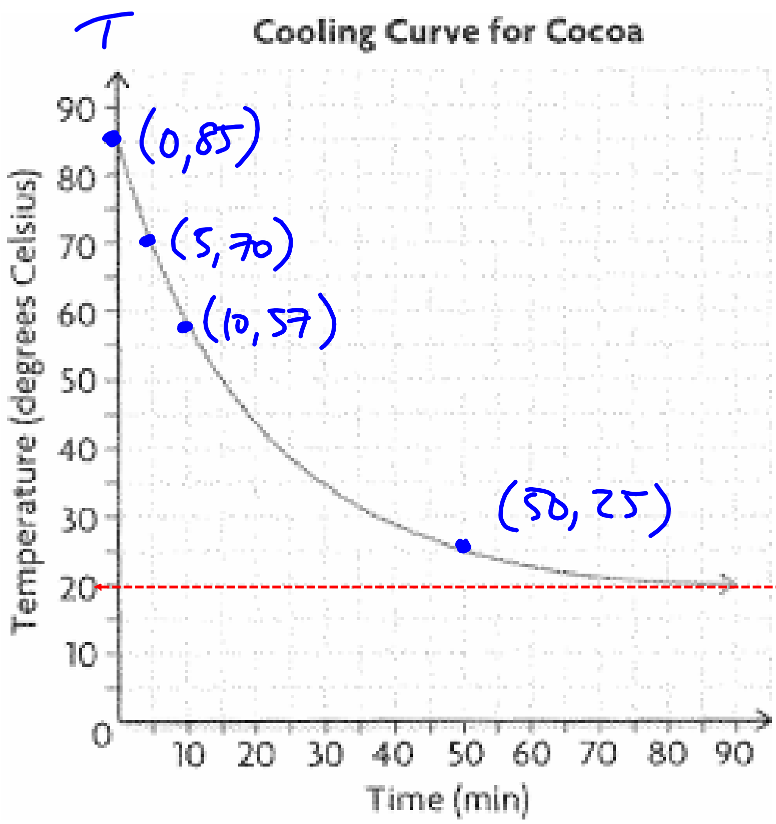
T



HA: T = 20

t

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a) What was the temperature at the start? $85°C$

b) What was the temperature after 1 hour? $23°C$

c) What was the temperature of the classroom? $20°C$

d) At what time was the cocoa 35°C? ~ 30 minutes

Apr 12-9:19 PM

e) Determine an algebraic model using *hours*:

$y = ab^x + q$

$q = 20$ (HA, room temperature)

$y = ab^x + 20$

recall: $y\text{-int} = a + q$

$85 = a + 20$

$65 = a$

$y = 65b^x + 20$

Sub any good point (y-int won't work)

$(60, 23) \longrightarrow (1, 23)$

min hours

$23 = 65b^1 + 20$

$3 = 65b$

$\frac{3}{65} = b$

In hours: (x or t)

$y = 65\left(\frac{3}{65}\right)^x + 20$

OR

$T = 65\left(\frac{3}{65}\right)^t + 20$

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f) Modify the algebraic model to use *minutes*:

nothing related to temperature has changed

$y = 65\, b^{x} + 20$

sub $(60, 23)$

$23 = 65\, b^{60} + 20$

$\frac{3}{65} = b^{60}$

$b = \pm \sqrt[60]{\frac{3}{65}}$, but $b > 0$

$y = \underset{a}{65}\left(\underset{b}{\sqrt[60]{\frac{3}{65}}}\right)^{x} + \underset{q}{20}$

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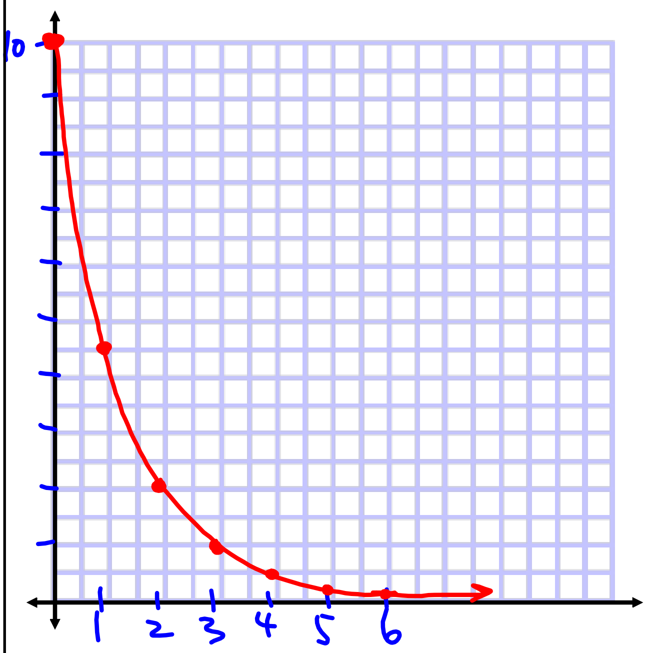
Ex.2 A tennis ball is dropped from 10 m. After each bounce, its height is 45% of the previous height.

a) Create a TOV and graph



# bounces	height (m)
0	10
1	4.5
2	2.025
3	0.911
4	0.41
5	0.18
6	0.08

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(b) Determine the equation that models the max height after n bounces.

h

$h(n) = ab^n + q$

$h(n) = 10b^n$

$h(n) = 10(0.45)^n$

HA: $y=0$ (floor)

$a = 10$ (y-int)

(c) Estimate the number of bounces required for the bounce height to be 10% or less of the starting height.

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Summary:

For <u>most</u> instances of exponential growth or decay, it is sufficient to work only with vertical transformations.

$$y = ab^x + q$$

Growth and decay are often expressed in terms of a <u>rate</u>, r, which may sometimes be a percentage.

<u>growth</u>	<u>decay</u>
$b = 1 + r$	$b = 1 - r$

100%

In many applications, the HA will be y=0 because it is the lowest possible value (e.g., populations, money, bouncing on a surface).

Exercises:

p.261 # [illegible], 4, 5, 7, 8, 9, 12, 13, 14, 15

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Exercises:

p.261 # 3, 4, 5, 7, 8, 9, 12, 13, 14, 15

d a

5. \$1000 , 6%/yr , 15 yrs.

$y = ab^x + q$

$f(x) = a\underbrace{b}^{x} + \underbrace{q}_{\text{often} = 0}$

$b = 1 \pm r$ $\quad r = \text{rate}$

100%

$b = 1 + 0.06$ $\quad q = 0$

$= 1.06$

$A(t) = a(1.06)^t$ $\quad A(0) = 1000$

$a = 1000$

$A(t) = 1000(1.06)^t$

(d) $A(15) = 1000(1.06)^{15}$

$\doteq 2396.56$

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13 (a) $b = 1 - 0.09$

$= 0.91$

$a = 100\%$

$= 1$

$I(d) = 1(0.91)^d + 0$

$I(d) = (0.91)^d$

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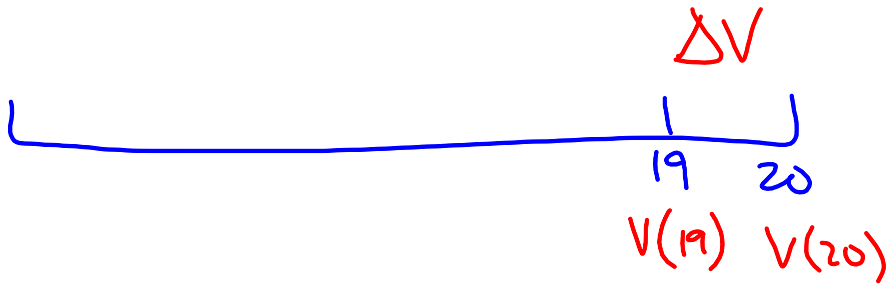
12(c) $V(t) = 5(1.06)^t$

$V(20) = 5(1.06)^{20}$



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14. $b = 1 - 0.99$

$= 0.01$ $\qquad g = 0$

$a = 100\%$

$G(n) = (0.01)^n$

(b) $G = 10^{10}$ $\qquad 0.01 = 10^{-2}$

$1 = 10^{10}(0.01)^n$

$\frac{1}{10^{10}} = (0.01)^n$

$10^{-10} = (10^{-2})^n$

$10^{-10} = 10^{-2n}$

$\Rightarrow -10 = -2n$

$n = 5 \rightarrow$ 1 bacterium remains

$\therefore$ 6 applications required

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