Exponential Applications (part 2) Nov. 5/2019 Doubling Period & Half-Life

Summary of Exponential Functions:

$$y = a(b)^{x} + q, b > 0, b \ne 1$$

where:

a is the scale factor

q is the lower or upper bound (asymptote)

a + q is the initial value/amount

x is the number of periods/cycles, elapsed time, etc.

y is the measured value after x

Exponential Growth: b > 1 Exponential Decay: 0 < b < 1

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Some relations occur so frequently that we have created special equations for them. It is not necessary to use these equations, but the benefit to using them is that the <u>common ratio</u> is known and does not need to be calculated.

<u>Doubling Period</u>: The time required for a quantity to grow to twice its original amount (b = 2).

The number of periods, x, becomes $\frac{t}{D}$

where *t* is the elapsed time, and *D* is the doubling period (i.e., the amount of time required to double the amount).

$$y = a(b)^x + q$$
 becomes $y = a(2)^{\frac{t}{D}} + q$



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<u>Half-Life</u>: The time required for a material to decay (or be reduced) to one-half of its original quantity (b = $\frac{1}{2}$).

The number of periods, x, becomes $\frac{t}{h}$

where *t* is the elapsed time, and *h* is the doubling period (i.e., the amount of time required to double the amount).

$$y = a(b)^x + q$$
 becomes $y = a(\frac{1}{2})^{\frac{t}{h}} + q$

Ex.1 The number of a certain bacteria doubles every 4 hours. The initial population is 36.

(a) Determine an exponential model for the number of bacteria after
$$t$$
 hours using $y = a(2)^{\frac{t}{D}} + q$
 $D = 4 \text{ hours}$
 $y = 36(2)^{\frac{t}{D}} + q$

with possible pop = 0

 $y = 36(2)^{\frac{t}{4}}$

(b) Determine the number of bacteria after 8 hours.

$$P(8) = 36(2)^{\frac{8}{4}}$$

= 36(4) : 144 bacteria
= 144 : after 8 hours.

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Ex.1 The number of a certain bacteria doubles every 4 hours. The initial population is 36.

(c) Determine an exponential model for the number of bacteria after t hours using $y = a(b)^x + q$

$$y = 36(2)^{\frac{t}{4}}$$

$$= 36(2^{\frac{t}{4}})^{t}$$

$$y = 36(1.189)^{t}$$

(d) Determine the number of bacteria after 8 hours. USW

$$y = 36(1.189)^{8}$$

= 143.8

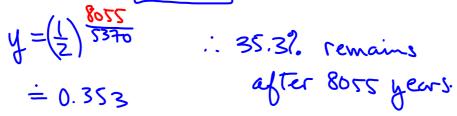
: 143 bacteria after & havs.

Ex.2 Archaeologists use the radioactive decay of carbon-14 to estimate the age of relics containing carbon. The half-life of carbon is 5370 years.

(a) Determine an exponential model for the amount of ¹⁴C present after *t* years (as compared to ¹²C) using

$$y = a(\frac{1}{2})^{\frac{t}{h}} + q$$
 $q = 0$
 $y = (\frac{1}{2})^{\frac{t}{5370}}$ $q = 0$
 $q = 0$

(b) Determine the percentage of ¹⁴C expected after 8055 years.



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Ex.2 Archaeologists use the radioactive decay of carbon-14 to estimate the age of relics containing carbon. The half-life of carbon is 5370 years.

(c) Determine an exponential model for the amount of ¹⁴C present after t years (as compared to ¹²C) using $v = a(b)^x + q$

(d) Determine the percentage of ¹⁴C expected after 8055 years

Exercises:

handout # 1-9

| 16
| 2a
| 9
| 3
| 5

1.(b)
$$N(d) = (25(2))$$

(a) Set $d = 0$, $N(0) = 125$

(b) Set $N(d) = 250$
 $250 = 125(2)^{\frac{d}{5}}$
 $2^{\frac{1}{25}} = 2^{\frac{d}{5}}$
 $2^{\frac{1}{25}} = 2^{\frac{d}{5}}$
 $2 = 2^{\frac{d}{5}}$
 $2 = 2^{\frac{d}{5}}$

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2 (a)
$$P(t) = 35000 \text{ b} + 9$$
 $9 = 0$

Sub $(10,44400)$
 $10\sqrt{44400} = 35000 \text{ b}^{10}$
 $10\sqrt{44400} = \text{b}$
 $10\sqrt{35000} = \text{b}$
 $10\sqrt{35000} = \text{b}$
 $10\sqrt{35000} = \text{b}$
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 $10\sqrt{3500} = 1.0$

3.
$$h = 2.5 \times 10^{5}$$
 $M_{0} = 1.0 \text{ g}$
 $m(t) = M_{0} \left(\frac{1}{2}\right)^{\frac{t}{h}}$

(a) $m(t) = 1\left(\frac{1}{2}\right)^{\frac{t}{2.5 \times 10^{5}}}$

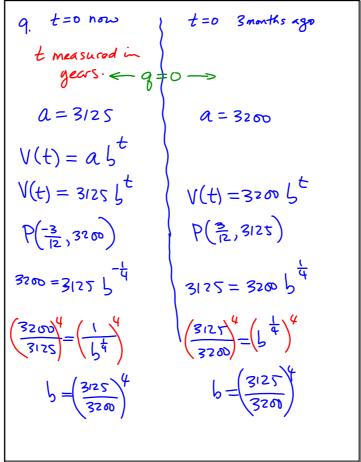
(b) $m(500) = \left(\frac{1}{2}\right)^{\frac{5000}{2.5 \times 10^{5}}}$

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5.
$$h = 2 \text{ grs}$$
 $m_0 = 5 \text{ hg}$
 $m(t) = m_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$
 $m(t) = 5 \left(\frac{1}{2}\right)^{\frac{t}{2}}$

(a) $m(t)$

(b) $l\text{Rmodhs} = 1.5 \text{ yrs}$
 $m(1.5)$



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