Exponential Applications (part 2) Doubling Period & Half-Life

Nov. 5/2019

Summary of Exponential Functions:

$$y = a(b)^x + q,\ b > 0,\ b \neq 1$$

where:

a is the scale factor
q is the lower or upper bound (asymptote)
$a + q$ is the initial value/amount
x is the number of periods/cycles, elapsed time, etc.
y is the measured value after x

Exponential Growth: b > 1   Exponential Decay: 0 < b < 1

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Some relations occur so frequently that we have created special equations for them. It is not necessary to use these equations, but the benefit to using them is that the common ratio is known and does not need to be calculated.

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Doubling Period: The time required for a quantity to grow to twice its original amount (b = 2).

The number of periods, x, becomes $\frac{t}{D}$

where t is the elapsed time, and D is the doubling period (i.e., the amount of time required to double the amount).

$y = a(b)^x + q$ becomes $y = a(2)^{\frac{t}{D}} + q$

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Half-Life: The time required for a material to decay (or be reduced) to one-half of its original quantity (b = $\frac{1}{2}$).

The number of periods, x, becomes $\frac{t}{h}$

where t is the elapsed time, and h is the doubling period (i.e., the amount of time required to double the amount).

$y = a(b)^x + q$ becomes $y = a(\frac{1}{2})^{\frac{t}{h}} + q$

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Ex.1 The number of a certain bacteria doubles every 4 hours. The initial population is 36.

(a) Determine an exponential model for the number of bacteria after *t* hours using $y = \underbrace{a}_{36}(2)^{\frac{t}{D}} + \underbrace{q}_{\text{min possible pop} = 0}$

D = 4 hours

$y = 36(2)^{\frac{t}{4}}$

OR

$P(t) = 36(2)^{\frac{t}{4}}$

(b) Determine the number of bacteria after 8 hours.

$P(8) = 36(2)^{\frac{8}{4}}$

$= 36(4)$

$= 144$

∴ 144 bacteria after 8 hours.

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Ex.1 The number of a certain bacteria doubles every 4 hours. The initial population is 36.

(c) Determine an exponential model for the number of bacteria after *t* hours using $y = a(b)^x + q$

$y = 36(2)^{\frac{t}{4}}$

$= 36\left[2^{\frac{1}{4}}\right]^t$

$y \doteq 36(1.189)^t$

(d) Determine the number of bacteria after 8 hours. using new model

$y = 36(1.189)^8$

$\doteq 143.8$

∴ 143 bacteria after 8 hours.

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Ex.2 Archaeologists use the radioactive decay of carbon-14 to estimate the age of relics containing carbon. The half-life of carbon is 5370 years.

(a) Determine an exponential model for the amount of ^{14}C present after *t* years (as compared to ^{12}C) using

$$y = a\left(\tfrac{1}{2}\right)^{\frac{t}{h}} + q$$

$q = 0$
$h = 5370$
$a = 100\%$
$= 1$

$$y = \left(\tfrac{1}{2}\right)^{\frac{t}{5370}}$$

(b) Determine the $\boxed{\text{percentage}}$ of ^{14}C expected after 8055 years.

$$y = \left(\tfrac{1}{2}\right)^{\frac{8055}{5370}}$$

$\doteq 0.353$

$\therefore$ 35.3% remains after 8055 years.

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Exercises:
handout # 1-9

1b
2a
9
3
5

1.(b) $N(d) = 125(2)^{\frac{d}{5}}$

(a) set $d=0$, $N(0)=125$

(b) set $N(d)=250$

$\frac{250}{125} = \frac{125}{125}(2)^{\frac{d}{5}}$

$2^1 = 2^{\frac{d}{5}}$

$\Rightarrow 1 = \frac{d}{5}$

$\boxed{d=5}$

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2(a) $P(t) = 35000\, b^t + q$

$q=0$

Sub $(10, 44400)$

$44400 = 35000\, b^{10}$

$\sqrt[10]{\ } = (\ )^{\frac{1}{10}}$

$\sqrt[10]{\frac{44400}{35000}} = b$

$b \doteq 1.02$ $\quad b = 1 \pm r$

$\therefore$ rate of increase is $\sim 2\%$/year.

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3. $h = 2.5 \times 10^5$

$m_0 = 1.0\ g$

$m(t) = m_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$

(a) $m(t) = 1\left(\frac{1}{2}\right)^{\frac{t}{2.5\times10^5}}$

(b) $m(5000) = \left(\frac{1}{2}\right)^{\frac{5000}{2.5\times10^5}}$

$0.5\ \boxed{y^x}\ (5000 \div 250000)$

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5. $h = 2$ yrs

$m_0 = 5\ kg$

$m(t) = m_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$

$m(t) = 5\left(\frac{1}{2}\right)^{\frac{t}{2}}$

(a) $m(4)$

(b) 18 months = 1.5 yrs

$m(1.5)$

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9. $t=0$ now

t measured in years. ← $q \neq 0$ →

$a = 3125$

$V(t) = a\,b^t$

$V(t) = 3125\,b^t$

$P\left(\frac{-3}{12}, 3200\right)$

$3200 = 3125\,b^{-\frac{1}{4}}$

$\left(\frac{3200}{3125}\right)^4 = \left(\frac{1}{b^{\frac{1}{4}}}\right)^4$

$b = \left(\frac{3125}{3200}\right)^4$

$t=0$ 3 months ago

$a = 3200$

$V(t) = 3200\,b^t$

$P\left(\frac{3}{12}, 3125\right)$

$3125 = 3200\,b^{\frac{1}{4}}$

$\left(\frac{3125}{3200}\right)^4 = \left(b^{\frac{1}{4}}\right)^4$

$b = \left(\frac{3125}{3200}\right)^4$

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