Unit Review

Topics:

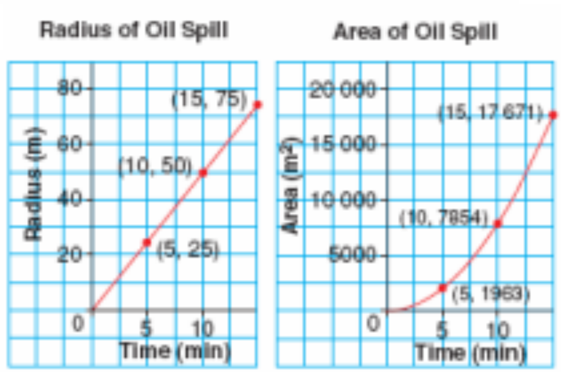
- ∞ Graphs of parent exponential functions & its key properties
- ∞ Graphs of exponential functions with different bases
- ∞ Transformed exponential functions
- ∞ Determining the equation of an exponential function
- ∞ Comparing linear, quadratic, and exponential functions
- ∞ Rates of change of exponential functions
- ∞ Transformations of exponential functions
- ∞ Laws of Exponents
- ∞ Solving exponential equations
- ∞ Problem solving with exponential functions (including doubling and half-life)

Questions to work on from the text:
Pg. 85 #1 -12, pg. 154 #1 – 4, 8, 9 *The textbook does not have a lot of graphing questions!

Supplementary problems:

Basic properties:

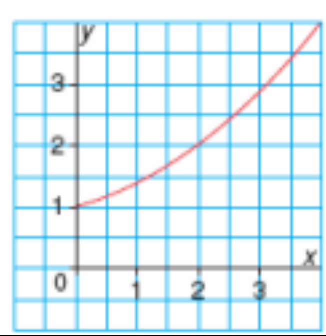
1. Which situation(s) represent exponential growth?
   a) A hairdresser increases the price of a haircut by $0.50 every year.
   b) Gua's investment doubles every 20 years.
   c) The players in each round of a tennis tournament are the winners from each pair in the previous round.
   d) Daryl makes $10/h working as a line chef.



2. A tanker runs aground, creating a circular oil spill.
   a) For each graph, calculate the average rate of change:
      i) From 0 min to 5 min
      ii) From 10 min to 15 min
      What do the rates of change represent in this situation?
   b) Describe the change in the radius of the spill.
   c) Describe the change in the area of the spill.

3. Does $y=5(1.08)^x$ increase or decrease? Justify your answer.

4. For the given curve, shown on the right, sketch a different curve with:
   a) A different starting value
   b) A greater rate of change
   c) A lesser rate of change



5. Sketch the graphs of $y=\left(\frac{1}{4}\right)^x$ and $y=\left(\frac{1}{5}\right)^x$ on the same set of axis.

6. Explain the restrictions on the base of the exponential function $(b>0, b\neq 1)$.

<u>Transformations</u>:

7. For the following functions, describe the transformations, write the function in exponential form and sketch the final graph. Describe the final function in term of the domain, range, increase/decrease and asymptote.

$y=-2f(-x)+1$ if i) $f(x)=2^x$ ii) $f(x)=3^x$ iii) $f(x)=\left(\frac{1}{2}\right)^x$

8. In each of the following cases state the parent function and describe the transformations it has undergone. Write the transformations in function notation, if it is not provided.

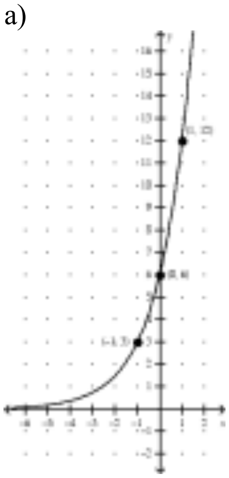
a) $y=-2^{x+3}$ b) $y=\left(\frac{1}{2}\right)^{2x}-1$ c) $y=2(5)^{-x}$

<u>Equation of Exponential functions</u>:

9. What is the equation of the function with horizontal asymptote $y = 2$, y-intercept = 10 and a common ratio of 3?

10. State the domain and range for each of the following functions, then determine the equations of the following exponential functions.

a)



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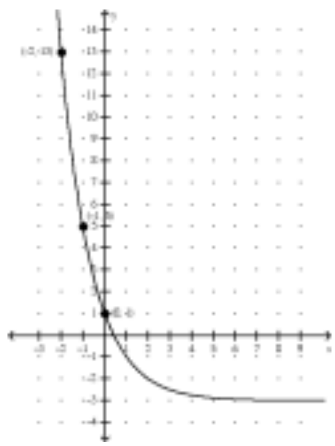
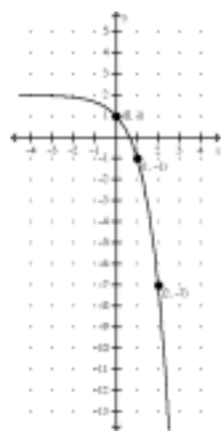


c)



<u>Exponent Laws</u>:

11. Simplify

a) $(-3)^{-3}$ b) $\left(\sqrt{2}x^{\frac{1}{2}}\right)^8$ c) $64^{\frac{5}{6}}$ d) $\left(a^{\frac{-1}{2}}\right)^{\frac{1}{2}}$

e) $\left(\frac{1}{8}\right)^{-5}$ f) $\left(\frac{-3}{4}\right)^3$ g) $1024^{\frac{-4}{5}}$ h) $\left(\frac{-8}{27}\right)^{\frac{-2}{3}}$ i) $(16x^8)^{\frac{3}{4}}$

12. Express as a single power

a) $2^{\frac{1}{3}}\cdot 2^{\frac{1}{3}}$ b) $\sqrt{\sqrt{\sqrt{x}}}$ c) $(2^{x+1})^{x-1}$ d) $\left(\sqrt[4]{x^3}\right)^2$

13. Evaluate.

a) $(-1)^3$ b) $(-1)^{304}$ c) -3^{-2} d) $\left(\frac{1}{2}\right)^{-1}$ e) $-(-1)^{253}$

Solving Equations:

14. Solve

a) $10^{3x-1}=1000$ b) $6^{5x-3}=1$ c) $2^{x+1}=16$ d) $2^{x^2+x}=4$

e) $2(3)^{x+1}=54$ f) $\left(\frac{1}{9}\right)^{x-1}=\left(\frac{1}{27}\right)^{x}$ g) $(2^x)(4^x)=16^x$ h) $5^{2x}-4(5^x)+3=0$

15. Determine the approximate solutions using your calculator. Round to two decimal places.

a) $3^x=30$ b) $5^x=10$ c) $5.6^y=60$ d) $(1.04)^x=2$

Applications:

16. A bacteria culture doubles in size every 15 minutes. How long will it take for a culture of 20 bacteria to grow to a population of 163 840?

17. Thorium-227 has a half-life of 18.4 days. How much time will a 50-mg sample take to decompose to 12.5 mg? To 10 mg?

18. A hot cup of coffee cools according to the equation $T(t)=69\left(\frac{1}{2}\right)^{\frac{t}{30}}+21$ where *T* is the temperature in degrees Celsius and *t* is the time in minutes.

a) Which part of the equation indicates that this is an example of exponential decay?
b) What was the initial temperature of the coffee?
c) Describe the transformations that the parent function has undergone.
d) Obtain the graph of this function from your teacher.
e) Determine the temperature of the coffee, to the nearest degree, after 48 min. (Do this both graphically and algebraically)
f) Explain how the equation would change if the coffee cooled faster.
g) Explain how the graph would change if the coffee cooled faster. Add your prediction to the graph.

19. The value of a car after it is purchased depreciates according to the formula $V(n)=28000(0.875)^n$.

a) What is the purchase price of the car?
b) What is the annual rate of depreciation? (i.e.: how much is the value of the car going down by?)
c) Obtain the graph of this function from your teacher.
d) What is the car's value at the end of 3 years? (Do this both graphically and algebraically)
e) What is the car's value at the end of 30 months? (Do this both graphically and algebraically)
f) How much value does the car lose in its first year?
g) How much value does it lose in its fifth year?

20. Write the equation that models each situation. In each case, describe each part of your equation.

a) the percent of a pond covered by water lilies if they cover one-third of a pond now and each week they increase their coverage by 10%
b) the amount remaining of the radioactive isotope U_{238} if it has a half-life of 4.5×10^9 years.
c) the intensity of light if each gel used to change the colour of a spotlight reduces the intensity of the light by 4%

21. The population of a city is growing at an average rate of 3% per year. In 1990, the population was 45 000.

a) Write an equation that models the growth of the city. Explain what each part of the equation represents.
b) Use your equation to determine the population of the city in 2007.
c) Determine the year during which the population will have doubled.
d) Suppose the population took only 10 years to double. What growth rate would be required for this to have happened?

Solutions for #9 - 21

9. $y = 8(3)^x + 2$

10. a) D = { $x \in R$ }, R = { $y \in R, y > 0$ }, $y = 6(2)^x$

b) D = { $x \in R$ }, R = { $y \in R, y > -3$ }, $y = 4\left(\frac{1}{2}\right)^x - 3$

c) D = { $x \in R$ }, R = { $y \in R, y < 2$ }, $y = -(3)^x + 2$

11. a) $-\frac{1}{27}$ b) $16x^4$ c) 32 d) $\sqrt[4]{\frac{1}{a}}$ e) 32768 f) $\frac{-27}{64}$

g) $\sqrt[5]{\left(\frac{1}{1024}\right)^4}$ or ≈ 0.00390625 h) -2.25 i) $8x^6$

12. a) $\sqrt[3]{4}$ b) $\sqrt[8]{x}$ c) 2^{x^2-1} d) $\sqrt{x^3}$

13. a) -1 b) 1 c) $-\frac{1}{9}$ d) 2 f) 1

14. a) $x = \frac{4}{3}$ b) $x = \frac{3}{5}$ c) $x = 3$ d) $x = 1, -2$ e) $x = 2$ f) $x = -2$ g) $x = 0$

h) $x = 0, x \simeq 0.68$

16. 195 minutes or 3 hours and 15 minutes

17. 36.8 days (12.5mg) and 42.5 days (approx, 10mg)

18. a) $b = \frac{1}{2}$ b) $90°C$ c) Vertical stretch by a factor of 69, Vertical Translation 21 units up. e) $44°C$ f) if $0 < b < \frac{1}{2}$

19. a) $28 000 b) 12.5% d) $18 757.81 e) $20 052.95

f) loses $3 500 g) loses $2 051.64

20. a) $y = \frac{1}{3}(1.1)^x$ b) $y = 100\left(\frac{1}{2}\right)^{\frac{x}{4.5 \times 10^9}}$ c) $y = 100(0.96)^x$

21. a) $y = 45000(1.03)^x$ b) 74 378 (approx.) c) by 2013 d) about 7%