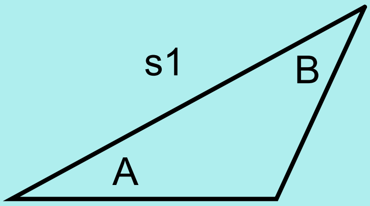
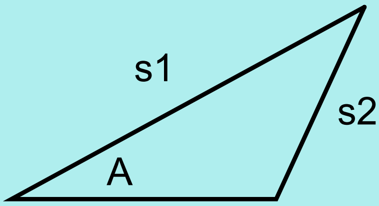
Recall: The Sine Law

The sine law is generally used when we have an oblique (non-right) triangle and:

(a) two angles and the enclosed side (ASA)



(b) two sides and a non-enclosed angle (SSA)



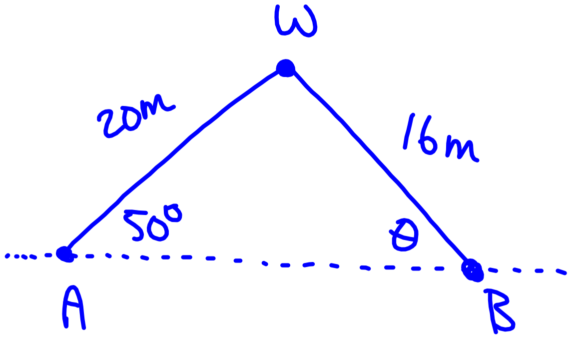
Apr 26-9:36 PM

Sine Law & Ambiguous Case

Nov. 14/2019

Ex.1 Albert and Belle are launching a weather balloon into a cloud. Albert's rope is 20 m long and makes an angle of 50° with the ground. Belle's rope is 16 m.

(a) Draw a labelled sketch and solve for the angle Belle's rope makes with the ground.



$$\frac{\sin\theta}{20} = \frac{\sin 50^\circ}{16}$$

$$\sin\theta = \frac{20\sin 50^\circ}{16}$$

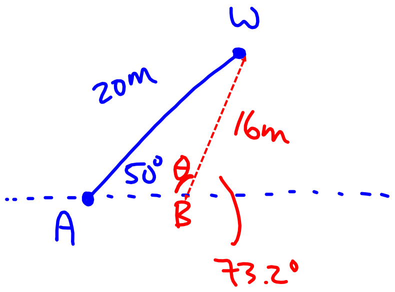
$$\theta \doteq 73.2^\circ$$

Nov 19-10:59 PM

Sine Law & Ambiguous Case

Ex.1 Albert and Belle are launching a weather balloon into a cloud. Albert's rope is 20 m long and makes an angle of 50° with the ground. Belle's rope is 16 m.

(a) Draw a second valid sketch and solve.



$\theta \doteq 180° - 73.2°$

$\doteq 106.8°$

$$\frac{\sin\theta}{20} = \frac{\sin 50°}{16}$$

$\theta \doteq 73.2°$

but θ is clearly obtuse

$\therefore \theta = 180° - 73.2°$

$\theta \doteq 106.8°$

Nov 19-10:59 PM

When we solve for <u>any angle</u> using the sine law, we must consider two possible solutions, one acute and one obtuse.

Common sense will often allow us to determine which answer is appropriate. For example:

- interior angles must add to 180°
- longer sides correspond to larger angles
- geometry of situation

With SSA, it is possible to encounter three situations:
(a) no solution - a triangle cannot be formed from the data
(b) one solution - a single triangle is possible
(c) two solutions - two <u>valid</u> triangles can be formed

For two solutions, must consider θ and $180° - \theta$

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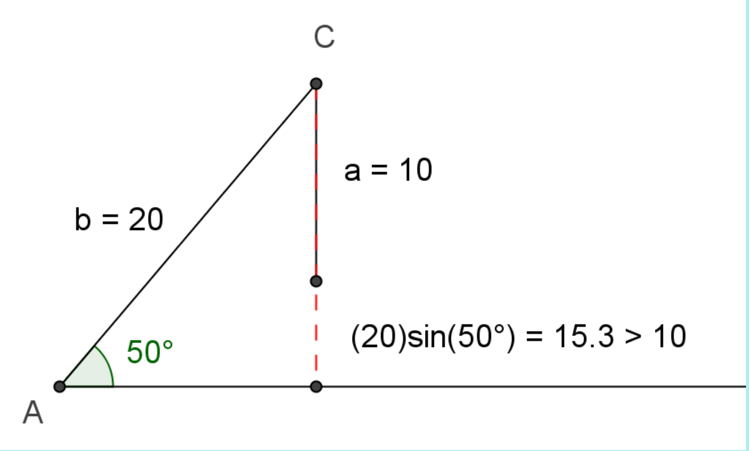
For acute angles (A < 90°):

Case 1: a < b sin A
too short, no triangle



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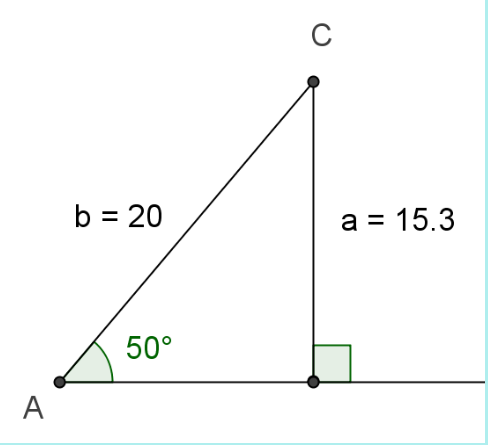
For acute angles (A < 90°):

Case 2: a = b sin A
right angle, one solution



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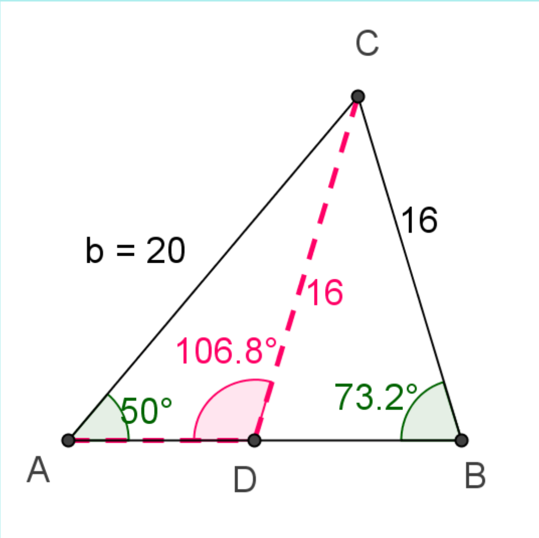
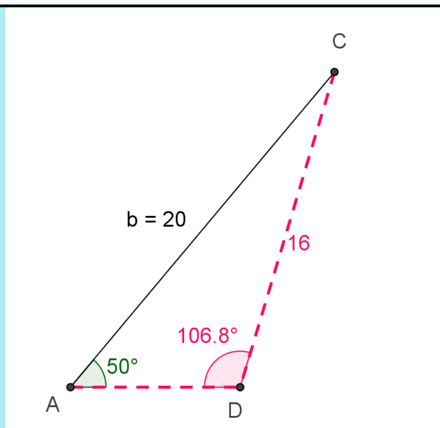
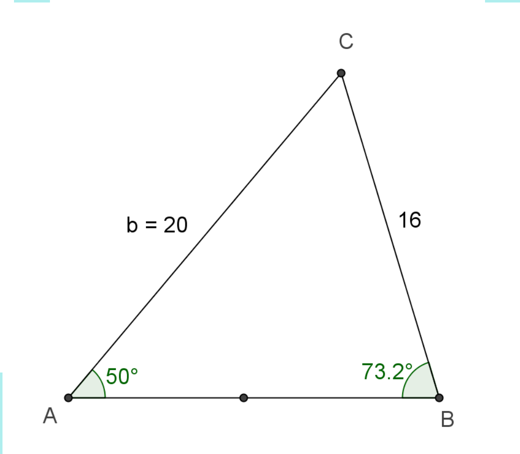
For acute angles (A < 90°):

Case 3: a > b sin A and a < b
two valid solutions







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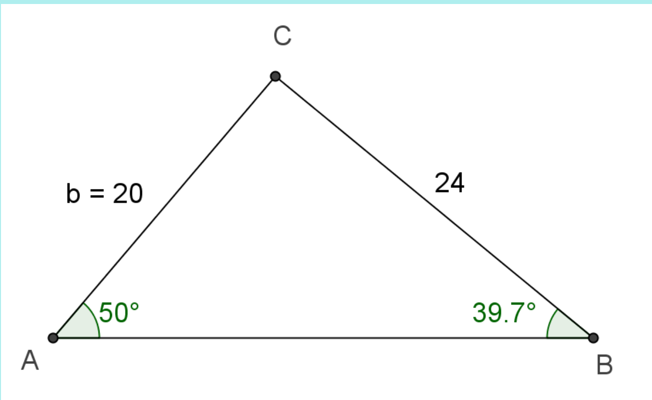
For acute angles (A < 90°):

Case 4: a $\geq$ b, one solution



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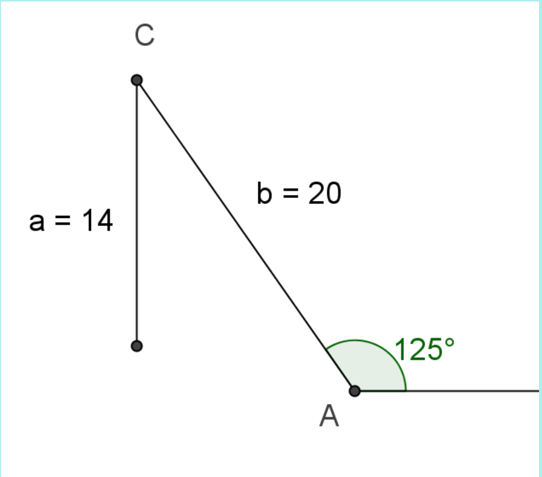
For obtuse angles (A > 90°):

Case 1: a $\leq$ b, too short, no triangle



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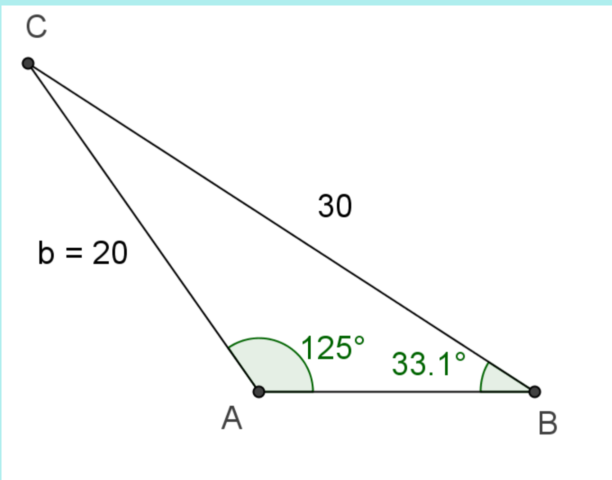
For obtuse angles (A > 90°):

Case 2: a > b, one solution



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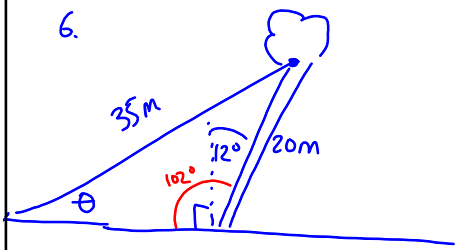
Assigned Work:

See summary, p.317

p.318 # 3, 5, 6, 7, 8, 10, 11

6.



$$\frac{\sin\theta}{20} = \frac{\sin 102^\circ}{35}$$

$$\sin\theta = \frac{20\sin 102^\circ}{35}$$

$\boxed{\theta \doteq 34^\circ}$ or $\theta \doteq 180^\circ - 34^\circ$

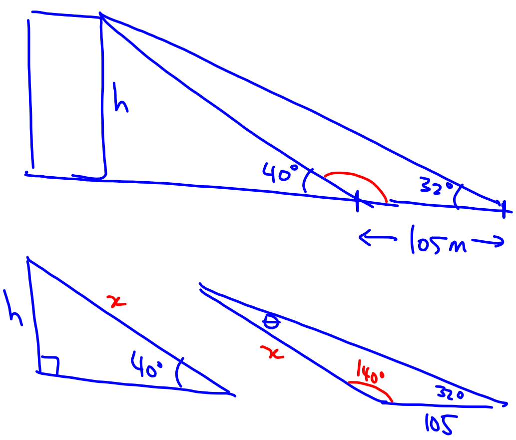
$\doteq 146^\circ$

but $146^\circ + 102^\circ > 180^\circ$

reject $\theta = 146^\circ$

Apr 21-12:17 AM

7.



$$\frac{x}{\sin 32^\circ} = \frac{105}{\sin 8^\circ}$$

$\theta = 180^\circ - 140^\circ - 32^\circ$

$= 8^\circ$

$$x = \frac{105\sin 32^\circ}{\sin 8^\circ}$$

$x \doteq$ ______

$$\sin 40^\circ = \frac{h}{x}$$

$x\sin 40^\circ = h$

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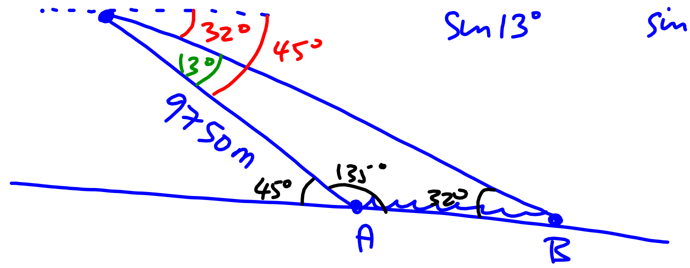
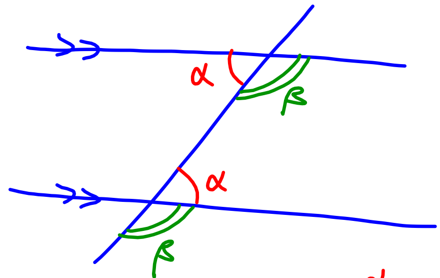
8.



$$\frac{AB}{\sin 13^\circ} = \frac{9750}{\sin 32^\circ}$$

$45^\circ - 32^\circ = 13^\circ$



$\alpha + \beta = 180^\circ$
(c-pattern)

$\beta = \beta$, F-pattern

$\alpha = \alpha$ z-pattern

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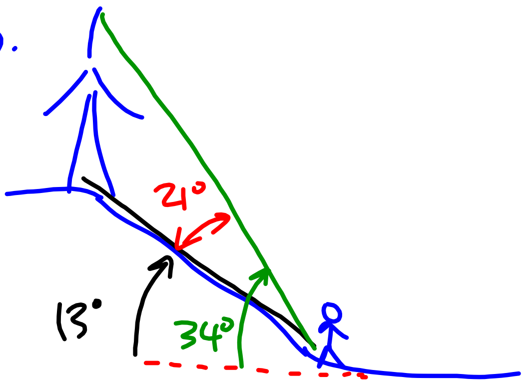
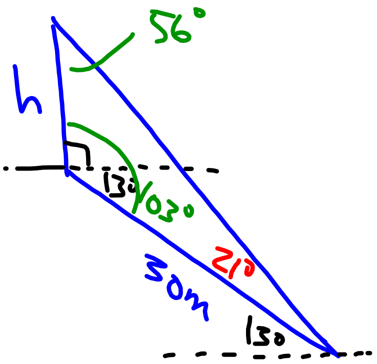
10.





$$\frac{h}{\sin 21^\circ} = \frac{30}{\sin 56^\circ}$$

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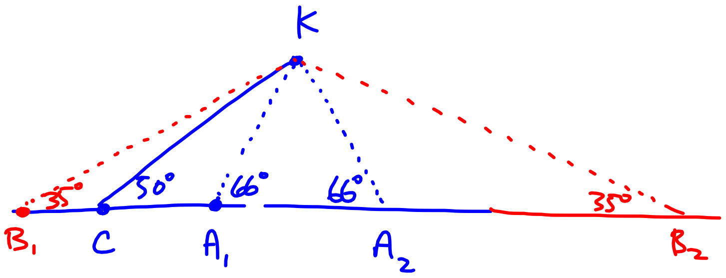
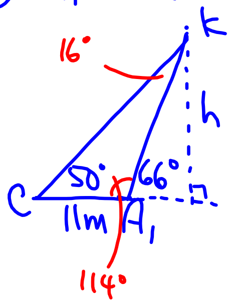
11.



① $CA_1 = 11\text{m}$



$$\frac{KA_1}{\sin 50°} = \frac{11}{\sin 16°}$$

$$KA_1 = \_\_\_\_\_\_\_\_$$

$$\sin 66° = \frac{h}{KA_1}$$

$$h = \_\_\_\_\_\_\_\_$$

Nov 18-1:07 PM