

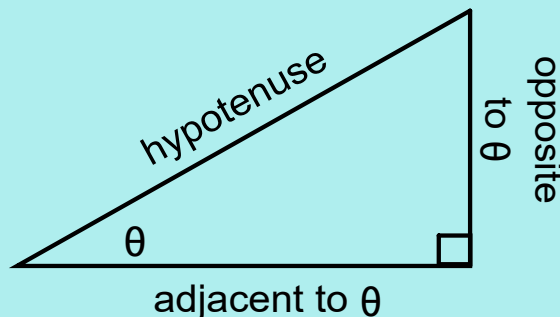
Recall:

For any angle of interest ( $\theta$ ), there are three (3) primary trigonometric ratios.

$$\text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

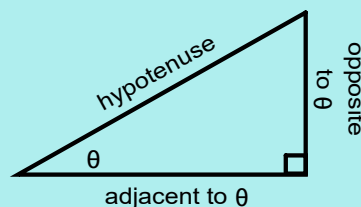
$$\text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent of } \theta = \frac{\text{opposite}}{\text{adjacent}}$$



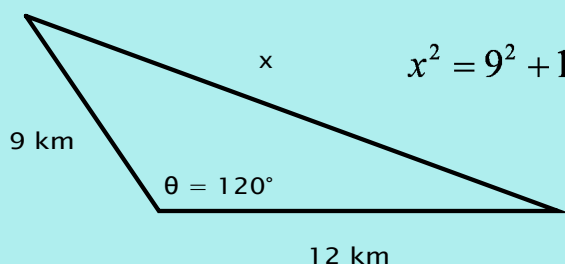
S o h C a h T o a

Our reference triangle (as shown) is generally represented as an acute triangle (i.e., all angles  $\leq 90^\circ$ ).



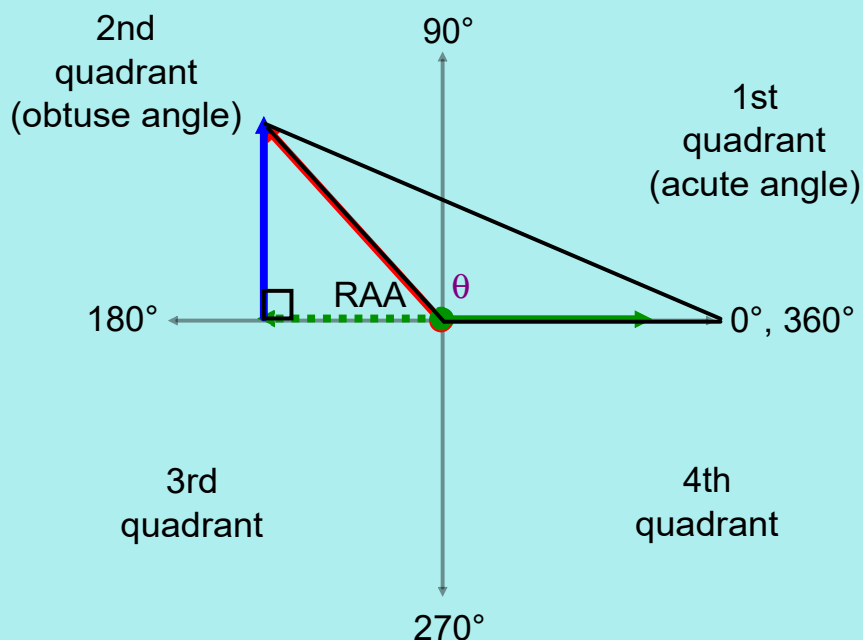
But...

using the cosine law, we have solved triangles such as the one shown below... how?



$$x^2 = 9^2 + 12^2 - 2(9)(12)\cos(120^\circ)$$

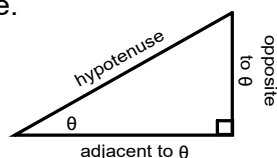
To work with angles greater than  $90^\circ$ , we form a right-triangle using the terminal arm and the related acute angle.



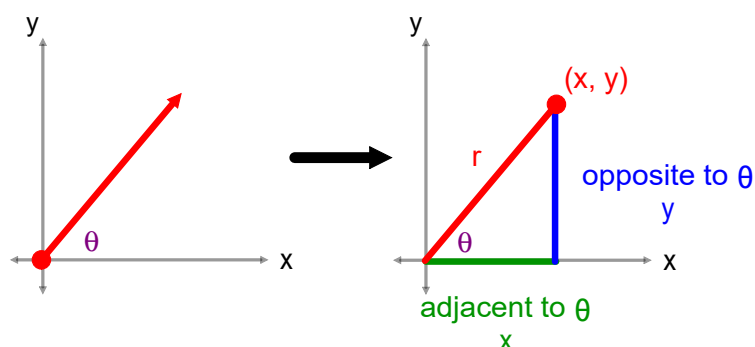
### Trigonometry of Obtuse Angles

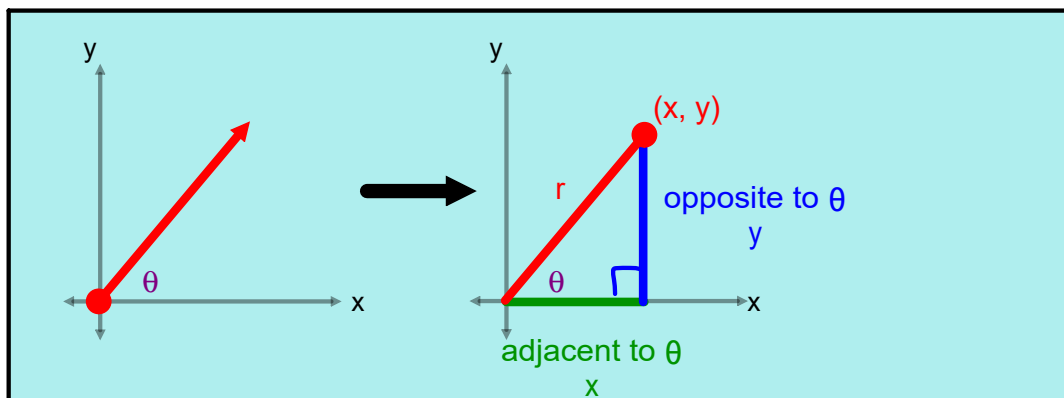
Nov. 19/2019

All trigonometric ratios are defined in terms of the sides of an acute right-triangle.



We can redefine the trig ratios for angles in standard position by drawing a right-triangle using the terminal arm.





where:  $r^2 = x^2 + y^2$

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$

This definition refers only to coordinates (x, y) and the distance (r) from the origin to the point, and should be valid for any angle.

Ex.1 The point (-4, 3) is on the terminal arm of an angle  $\theta$  in standard position. Find sine and cosine for  $\theta$ .



$$\begin{aligned} r^2 &= 3^2 + 4^2 \\ r^2 &= 9 + 16 \\ r &= 5, r > 0 \end{aligned}$$

$$\sin RAA = \frac{3}{5} \quad \cos RAA = \frac{4}{5} \quad \tan RAA = \frac{3}{4}$$

$$\sin \theta = \frac{3}{5} \quad \cos \theta = -\frac{4}{5} \quad \tan \theta = \frac{3}{-4}$$

$$RAA = \sin^{-1}\left(\frac{3}{5}\right)$$

$$\hat{=} 36.9^\circ$$

$$\theta = \sin^{-1}\left(\frac{3}{5}\right)$$

$$\hat{=} 36.9^\circ$$

$$\begin{aligned} \text{OR } \theta &\hat{=} 180^\circ - 36.9^\circ \\ &= 143.1^\circ \\ &\text{(ambiguous case)} \end{aligned}$$

reject  $36.9^\circ$   
because P(-4, 3) in Q2.

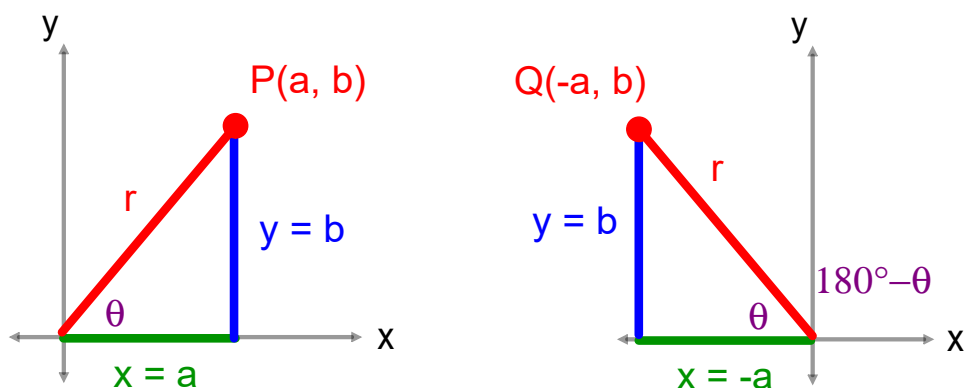
$$\cos RAA = \frac{4}{5}$$

$$RAA \hat{=} 36.9^\circ$$

$$\cos \theta = -\frac{4}{5}$$

$$\theta \hat{=} 143.1^\circ$$

Focusing on obtuse angles ( $90^\circ < \theta < 180^\circ$ ), consider the points  $P(a, b)$  and  $Q(-a, b)$ .

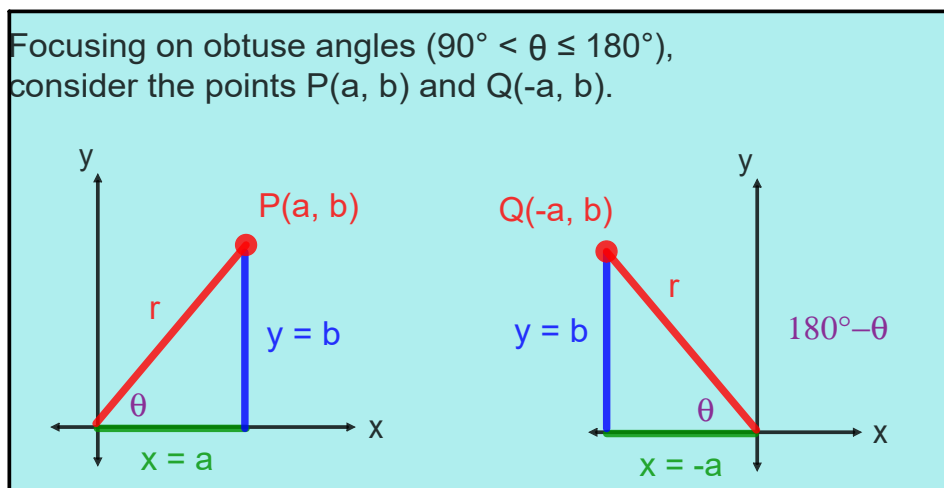


How can we relate  $\sin \theta$  to  $\sin(180^\circ - \theta)$ ?

$$\sin \theta = \sin(180^\circ - \theta)$$

$$\text{e.g., } \sin 30^\circ = \sin 150^\circ$$

$$\sin 70^\circ = \sin 110^\circ$$



How can we relate  $\cos \theta$  to  $\cos(180^\circ - \theta)$ ?

$$\cos \theta = -\cos(180^\circ - \theta)$$

$$\text{or } -\cos \theta = \cos(180^\circ - \theta)$$

$$\text{e.g., } \cos 30^\circ = -\cos 150^\circ$$

$$-\cos 45^\circ = \cos 135^\circ$$

Ex.2 Express each of the following in terms of the related acute angle, then confirm your answer.

(a)  $\sin(125^\circ)$

$$= \sin(180^\circ - 125^\circ)$$
$$= \sin(55^\circ)$$

(b)  $\cos(160^\circ)$

$$= -\cos(180^\circ - 160^\circ)$$
$$= -\cos(20^\circ)$$

Ex.3 Find  $\theta$ , if  $0 \leq \theta \leq 180^\circ$ .

(a)  $\sin \theta = 0.25$

$$\theta = \sin^{-1}(0.25)$$
$$\doteq 14.5^\circ$$

or

$$\theta \doteq 180^\circ - 14.5^\circ$$
$$= 165.5^\circ$$

(b)  $\cos \theta = -0.87$

$$\theta = \cos^{-1}(-0.87)$$
$$\theta \doteq 150.5^\circ$$

Assigned Work:

Handout (from old text, p.281)

# 1, 2odd (express in terms of RAA first)

3odd, 5, 6, 9, 12\*