

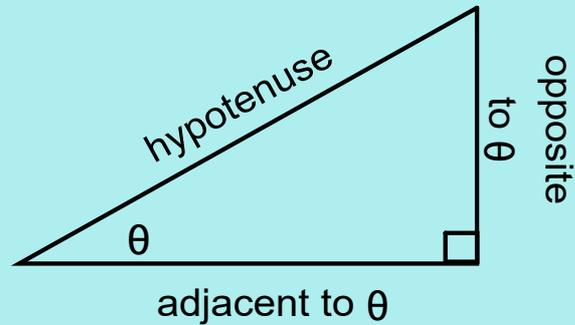
Recall:

For any angle of interest (θ), there are three (3) primary trigonometric ratios.

$$\text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

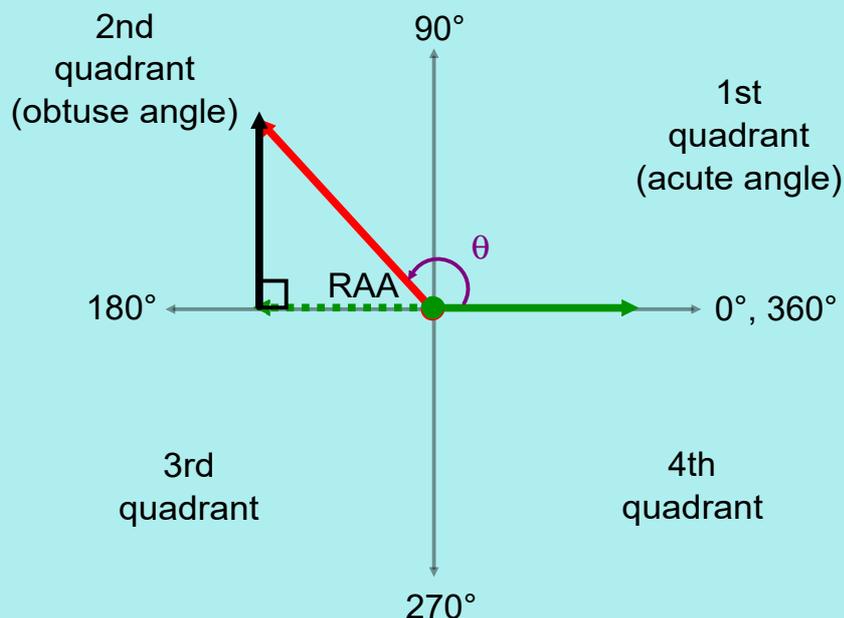
$$\text{tangent of } \theta = \frac{\text{opposite}}{\text{adjacent}}$$



S o h C a h T o a

Apr 25-9:54 PM

To work with angles greater than 90° , we form a right-triangle using the terminal arm and the related acute angle.



Apr 19-9:19 PM

Trigonometry of Any Angle: The CAST Rule

Nov. 20/2019

Any angle in standard position has a related acute angle.
A right-triangle can always be drawn using this
RAA.

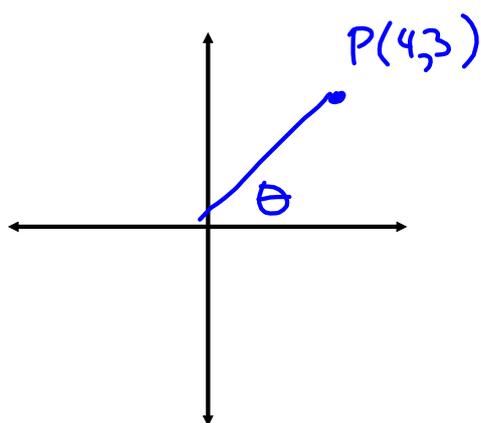
Therefore any angle can be associated with the
primary trig ratios.

The quadrant will determine the sign of the ratio.

Apr 19-9:13 PM

Ex. Determine the primary trigonometric ratios for:

(a) P(4, 3)



$$r^2 = 3^2 + 4^2$$

$$r^2 = 25$$

$$r = 5, r > 0$$

$$\sin \theta = \frac{y}{r} = \frac{3}{5}$$

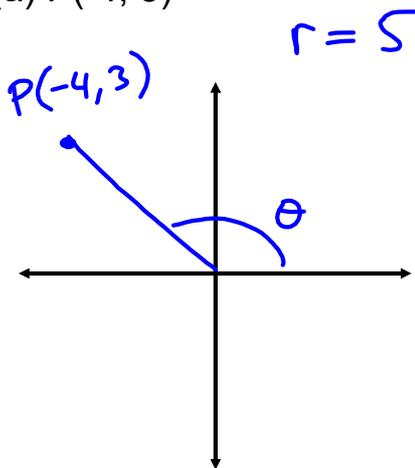
$$\cos \theta = \frac{x}{r} = \frac{4}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{3}{4}$$

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Ex. Determine the primary trigonometric ratios for:

(a) P(-4, 3)



$$\sin \theta = \frac{3}{5}$$

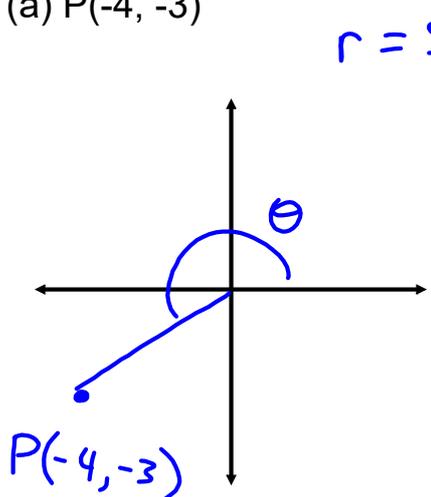
$$\cos \theta = \frac{-4}{5}$$

$$\begin{aligned} \tan \theta &= \frac{3}{-4} \\ &= -\frac{3}{4} \end{aligned}$$

Apr 19-9:13 PM

Ex. Determine the primary trigonometric ratios for:

(a) P(-4, -3)



$$\sin \theta = \frac{-3}{5} \quad \cancel{\frac{3}{5}}$$

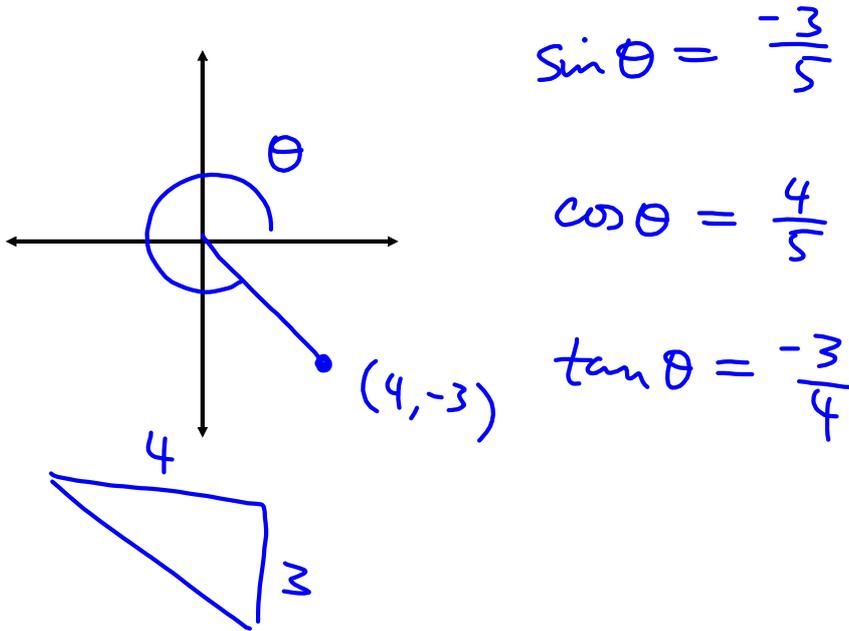
$$\cos \theta = \frac{-4}{5} \quad \cancel{\frac{4}{5}}$$

$$\begin{aligned} \tan \theta &= \frac{-3}{-4} \\ &= \frac{3}{4} \quad \cancel{\frac{3}{4}} \end{aligned}$$

Apr 19-9:13 PM

Ex. Determine the primary trigonometric ratios for:

(a) P(4, -3)



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The CAST rule allows us to quickly determine the sign of each trig ratio for any quadrant.

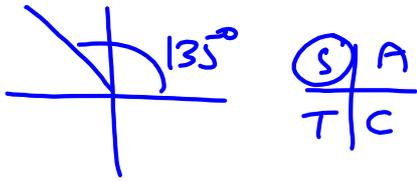
| | | | |
|----|-------------------------|----|-------------------------|
| Q2 | sin + cos - tan - | Q1 | sin + cos + tan + |
| Q3 | sin - cos - tan + | Q4 | sin - cos + tan - |

| | |
|--------------|--------|
| Sine S | All |
| T tangent | Cosine |

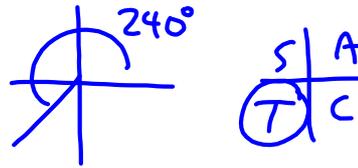
May 3-9:19 AM

Ex.2 Predict the sign of each value (verify with calculator)

(a) $\tan 135^\circ < 0$

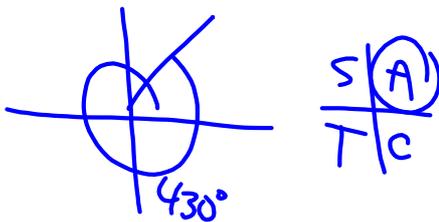


(b) $\cos 240^\circ < 0$

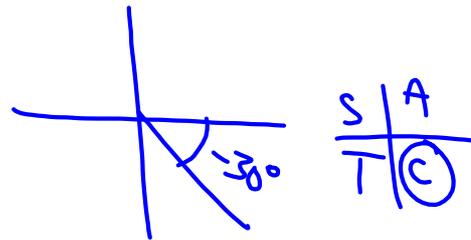


(c) $\sin 430^\circ > 0$

$= \sin 70^\circ$



(d) $\tan(-30^\circ) < 0$



May 3-9:22 AM

Ex.3 For $\tan \theta = -\frac{5}{24}$, where $0^\circ \leq \theta < 360^\circ$

(a) where (which quadrant) is θ ?

① $\frac{S}{T}$ $\frac{A}{C}$

Q2
or
Q4

② $\tan \theta = \frac{y}{x}$

Q4
 $\frac{y}{x} = \frac{-5}{24}$

OR
 $\frac{y}{x} = \frac{5}{-24}$
Q2

(b) determine the primary trig ratios

$r^2 = x^2 + y^2$

$r^2 = (-5)^2 + 24^2$

$r^2 = 601$

$r = \sqrt{601}, r > 0$

Q2
 $x = -24, y = 5$

$\sin \theta = \frac{5}{\sqrt{601}}$

$\cos \theta = \frac{-24}{\sqrt{601}}$

$\frac{S}{T}$ $\frac{A}{C}$

Q4
 $x = 24, y = -5$

$\sin \theta = \frac{-5}{\sqrt{601}}$

$\cos \theta = \frac{24}{\sqrt{601}}$

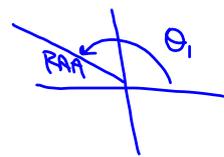
May 3-9:24 AM

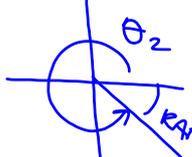
Ex.3 For $\tan \theta = -\frac{5}{24}$, where $0^\circ \leq \theta \leq 360^\circ$

(c) determine the value(s) of θ to the nearest degree

① $RAA = \tan^{-1}\left(\frac{5}{24}\right)$ *ignore any negative*
 $RAA \doteq 11.7683^\circ$

② $\frac{S}{T} \frac{A}{C}$ Q2, Q4

③  $\theta_1 = 180^\circ - RAA$
 $\doteq 168.2^\circ$

 $\theta_2 = 360^\circ - RAA$
 $\doteq 348.2^\circ$

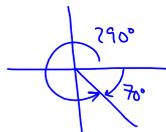
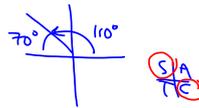
May 3-9:24 AM

Assigned Work:

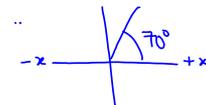
p.299 # 1, 2, 4, 6, 8, 9, 10, 12

$$\frac{c}{a} = \frac{e}{b}$$

4(c) $\tan 110^\circ$
 $= -\tan 70^\circ$
 $= \tan 290^\circ$

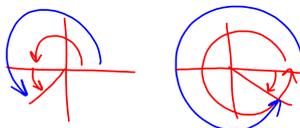
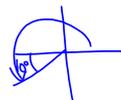


$RAA = 70^\circ$



4(d) $\sin 350^\circ$
 $= -\sin 10^\circ$
 $= \sin 190^\circ$

$RAA = 10^\circ$

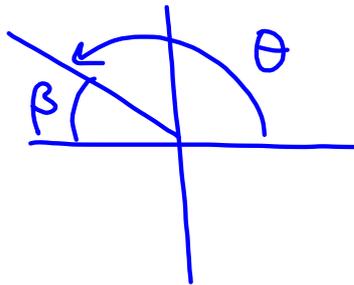


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$$6(c) \quad \cos \theta = -\frac{1}{4} \quad Q2$$

$$(iii) \quad RAA = \cos^{-1}\left(\frac{1}{4}\right)$$

$$\beta = \underline{\hspace{2cm}}$$



$$\theta = 180^\circ - \beta$$

$$= \underline{\hspace{2cm}}$$

Nov 21-2:36 PM

8 bdc

$$(b) \quad \tan \theta = -0.1623$$

$$\textcircled{1} \quad RAA = \tan^{-1}(0.1623)$$

$$\textcircled{2} \quad \begin{matrix} \text{S} \\ \text{A} \\ \text{T} \\ \text{C} \end{matrix}$$

$$\textcircled{3} \quad \theta_1 = 180^\circ - RAA$$

$$\theta_2 = 360^\circ - RAA$$

Q2:



Q4:



$$(d) \quad \cot \theta = 8.1516$$

$$\frac{1}{\tan \theta} = 8.1516$$

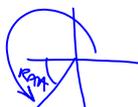
$$\frac{1}{8.1516} = \tan \theta$$

$$\textcircled{1} \quad RAA = \tan^{-1}\left(\frac{1}{8.1516}\right) = 6.994^\circ$$

$$\textcircled{2} \quad \begin{matrix} \text{S} \\ \text{A} \\ \text{T} \\ \text{C} \end{matrix}$$

$$\textcircled{3} \quad Q1: \theta_1 = 6.994^\circ$$

$$Q3: \theta_2 = 180^\circ + RAA = 186.994^\circ$$



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12. $\cos \theta = -\frac{5}{12} = \frac{x}{r}$ $r > 0$, length of the terminal arm.

(a) $\frac{S}{A}$ Q2 or Q3

(b) $r = 12$ $x = -5$

$$x^2 + y^2 = r^2$$

$$25 + y^2 = 144$$

$$y = \pm \sqrt{119}$$

$\sin \theta = \frac{\sqrt{119}}{12}$ or $\sin \theta = -\frac{\sqrt{119}}{12}$

$\tan \theta = \frac{\sqrt{119}}{-5}$ or $\tan \theta = -\frac{\sqrt{119}}{5}$

$= -\frac{\sqrt{119}}{5}$ $= \frac{\sqrt{119}}{5}$

(c) $RAA = \cos^{-1}\left(\frac{5}{12}\right)$

$=$ _____

Q2 $\theta_1 = 180^\circ - RAA$

Q3 $\theta_2 = 180^\circ + RAA$

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WS #11.

$$\tan \theta = -1.6003$$

$$\textcircled{1} RAA = \tan^{-1}(1.6003)$$

$$\doteq 58^\circ$$

$$\textcircled{2} \frac{S}{A}$$

$$\textcircled{3} Q2: \theta = 180^\circ - RAA$$

$$\doteq 22^\circ$$

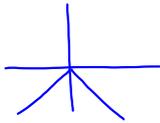
$$Q4: \theta = 360^\circ - RAA$$

$$\doteq 302^\circ$$

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WS
7. $\sin \theta = \frac{-4}{11}$ $\frac{y}{r}$ $r=11$
 $y=-4$

$x^2 + y^2 = r^2$
 $x^2 + (-4)^2 = 11^2$
 $x^2 = 105$
 $x = \pm\sqrt{105}$



$\cos \theta = \frac{\pm\sqrt{105}}{11}$

$\tan \theta = \frac{-4}{\pm\sqrt{105}}$

$= \pm \frac{4}{\sqrt{105}} \times \frac{\sqrt{105}}{\sqrt{105}}$

$= \pm \frac{4\sqrt{105}}{105}$

Nov 22-12:51 PM