

Solving Trigonometric Equations

To solve a trigonometric equation means to find the angle value(s) that satisfy the given equation.

recall: solving linear equations

Solve for x:

$$2x + 3 = 4x - 5$$

$$P = 2x$$

$$x = 4$$

recall: related acute angles

recall: CAST Rule

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Solving Trigonometric Equations

To solve a trigonometric equation means to find the angle value(s) that satisfy the given equation.

Steps:

1. The equation should involve only one trigonometric ratio.
2. Isolate the trigonometric ratio.
3. Solve for the related acute angle (using positive ratio).
4. Use the actual sign of the ratio to determine the quadrant(s) for your answer [CAST].
5. Determine the angles, within your chosen quadrants, using the related acute angle.
6. Determine any additional coterminal angles, if necessary.

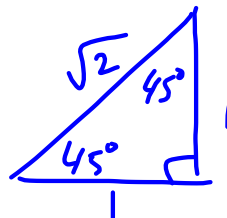
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Solving Trigonometric Equations

Ex.1 Solve

(a) $\cos \theta = -\frac{\sqrt{2}}{2}$ where $0^\circ \leq \theta < 360^\circ$

① RAA = $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$
 $= 45^\circ$



recall:

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

② $\frac{S}{T} = \frac{A}{C}$

③ Q2: $\theta = 180^\circ - \text{RAA}$
 $= 135^\circ$

Q3: $\theta = 180^\circ + \text{RAA}$
 $= 225^\circ$

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b) $\sin \theta + \sqrt{3} = -\sin \theta$ where $0^\circ \leq \theta < 720^\circ$

$$\frac{2 \sin \theta}{2} = \frac{-\sqrt{3}}{2}$$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$x + \sqrt{3} = -x$$

$$2x = -\sqrt{3}$$

$$x = -\frac{\sqrt{3}}{2}$$

① RAA = $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
 $= 60^\circ$



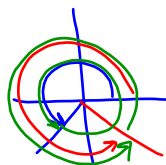
② $\frac{S}{T} = \frac{A}{C}$ ③ Q3: $\theta = 180^\circ + \text{RAA}$
 $= 240^\circ$

Q4: $\theta = 360^\circ - \text{RAA}$
 $= 300^\circ$

$0^\circ \leq \theta < 720^\circ$, need coterminal angles.

$$240^\circ + 360^\circ = 600^\circ$$

$$300^\circ + 360^\circ = 660^\circ$$



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d) $3\sin(2\theta) + 3 = 5$ where $0^\circ \leq \theta < 360^\circ$

$3\sin x + 3 = 5$ let $x = 2\theta$

$3\sin x = 2$

$\sin x = \frac{2}{3}$

① RAA = $\sin^{-1}\left(\frac{2}{3}\right)$
 $\approx 41.8^\circ$

② $\frac{S}{T/C}$ ③ Q1: $x \approx 41.8^\circ$
 Q2: $x \approx 180^\circ - \text{RAA}$
 $\approx 138.2^\circ$

$(0^\circ \leq \theta < 360^\circ) \times 2$ $x_1 \approx 41.8^\circ$
 $0^\circ \leq 2\theta < 720^\circ$ $x_2 \approx 138.2^\circ$
 $0^\circ \leq x < 720^\circ$ $x_3 \approx x_1 + 360^\circ$
 $\approx 401.8^\circ$
 but $x = 2\theta$ $x_4 \approx x_2 + 360^\circ$
 $\approx 498.2^\circ$
 $\theta = \frac{x}{2}$

$\theta_1 \approx 20.9^\circ$ $\theta_2 \approx 69.1^\circ$
 $\theta_3 \approx 200.9^\circ$ $\theta_4 \approx 249.1^\circ$

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d) $\cos \theta = 2\sin \theta \cos \theta$ where $0^\circ \leq \theta < 360^\circ$

$0 = 2\sin \theta \cos \theta - \cos \theta$ $x = 2xy$

$0 = \cos \theta (2\sin \theta - 1)$ $0 = 2xy - x$

$0 = x(2y - 1)$

$\cos \theta = 0$ $2\sin \theta - 1 = 0$ $x = 0$ $2y - 1 = 0$

$\frac{x}{r} = 0$ $2\sin \theta = 1$ $2y = 1$

$x = 0$ $\sin \theta = \frac{1}{2}$ $y = \frac{1}{2}$

$90^\circ (0, y)$ ① RAA = 30°


$(0, -y)$ ② $\frac{S}{T/C}$

270° ③ Q1: $\theta_1 = 30^\circ$

Q2: $\theta_2 = 180^\circ - 30^\circ$
 $= 150^\circ$

$\theta_3 = 90^\circ$

$\theta_4 = 270^\circ$



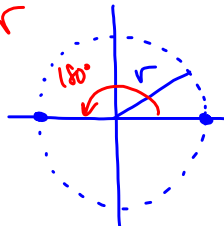
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Assigned Work:

- Handout # 2, 5ab (linear equations)
 # 14ab (linear, challenging)
 # 3ace, 5ce, 11 (non-linear, challenging)

14(b) $\cos 2x = -1$
 $\cos \theta = -1 = \frac{x}{r}$
 $\theta_1 = 180^\circ$

$r = -x$
 $r > 0$



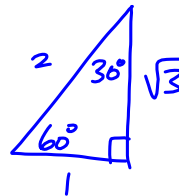
$0^\circ \leq x \leq 360^\circ$
 $0^\circ \leq 2x \leq 720^\circ$
 $0^\circ \leq \theta \leq 720^\circ$

$\theta_1 = 180^\circ$
 $2x_1 = 180^\circ$
 $x_1 = 90^\circ$

$\theta_2 = 180^\circ + 360^\circ$
 $= 540^\circ$
 $2x_2 = 540^\circ$
 $x_2 = 270^\circ$

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14(i) $2 \sin(0.5x) = 1$
 $2 \sin \theta = 1$
 $\sin \theta = \frac{1}{2}$



① RAA = 30°
 ② $\frac{S}{T} = \frac{A}{C}$ ③ Q1: $\theta_1 = 30^\circ$
 Q2: $\theta_2 = 180^\circ - \text{RAA}$
 $= 150^\circ$

$(0^\circ \leq x \leq 360^\circ) \times 0.5$
 $0^\circ \leq 0.5x \leq 180^\circ$
 $0^\circ \leq \theta \leq 180^\circ$

$0.5x_1 = 30^\circ$
 $x_1 = 60^\circ$
 $0.5x_2 = 150^\circ$
 $x_2 = 300^\circ$

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$$3 (a) \quad 2\cos^2 x - 7\cos x + 3 = 0$$

$$2a^2 - 7a + 3 = 0$$

$$(2a-1)(a-3) = 0$$

$$a = \frac{1}{2}$$

$$a = 3$$

M 6

A -7

N $\frac{-1}{2}, \frac{-6}{2}$

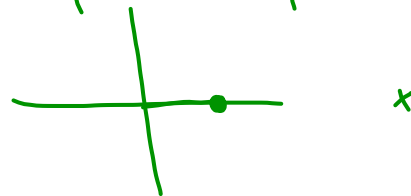
$$\cos x = \frac{1}{2}$$

$$\cos x = \frac{3}{1} = \frac{x}{r} \quad \text{not possible}$$

① RAA

② CAST

③ Q's.



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$$11. (b) \quad \cos x \tan x - 1 + \tan x - \cos x = 0$$

$$\underbrace{\cos x \tan x + \tan x} - \underbrace{\cos x - 1} = 0$$

$$\tan x (\underbrace{\cos x + 1}) - 1 (\underbrace{\cos x + 1}) = 0$$

$$(\cos x + 1)(\tan x - 1) = 0$$

$$\cos x = -1 = \frac{x}{r}$$

① RAA

②

③

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