Unit 6 - Periodic Functions

Nov. 27/2019

A periodic function is a function that repeats its behaviour identically at regular intervals. The pattern that repeats is called a cycle.

The length of one cycle (measured along the horizontal axis) is called the period of the function.

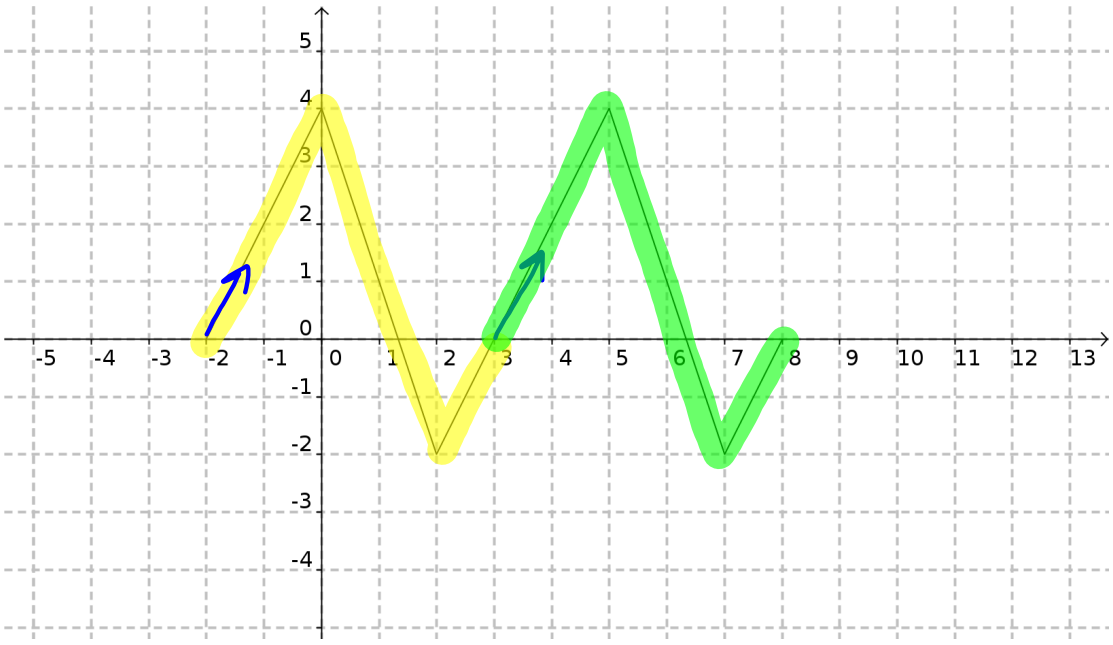
The horizontal line halfway between the maximum and minimum value is called the axis of the curve, which has the equation:

$$y = \frac{maximum + minimum}{2}$$

The vertical distance from the axis of the curve to the maximum (or minimum) is called the amplitude. The amplitude can be determined using the equation:

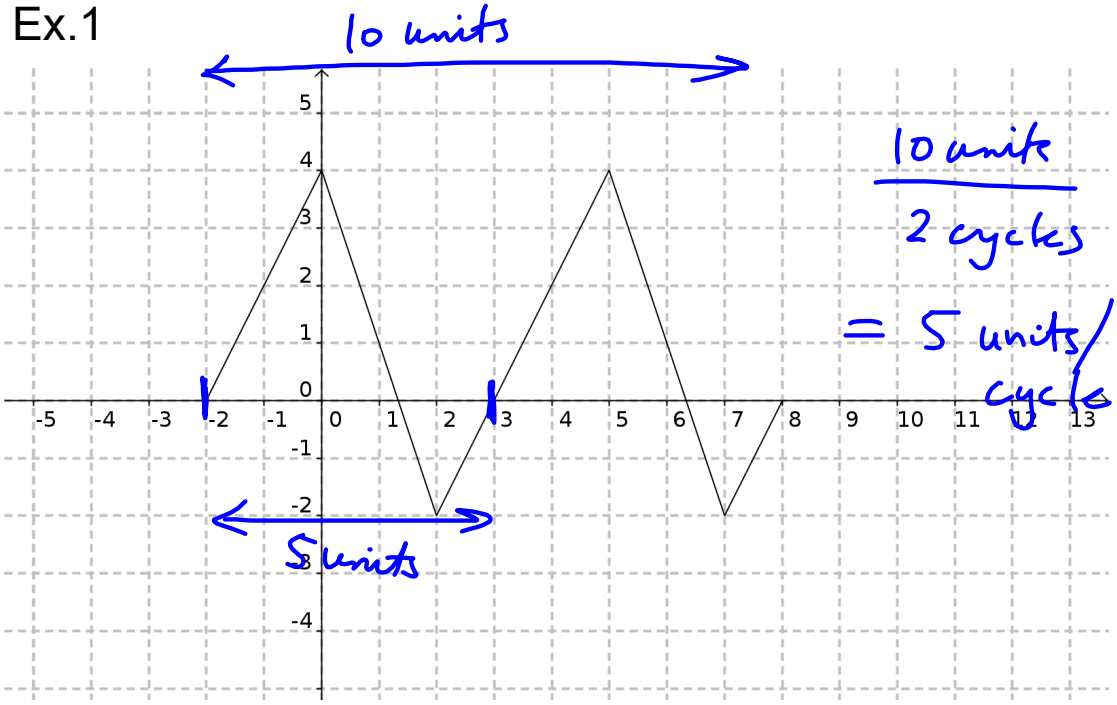
$$a = \frac{maximum - minimum}{2}$$

Ex.1



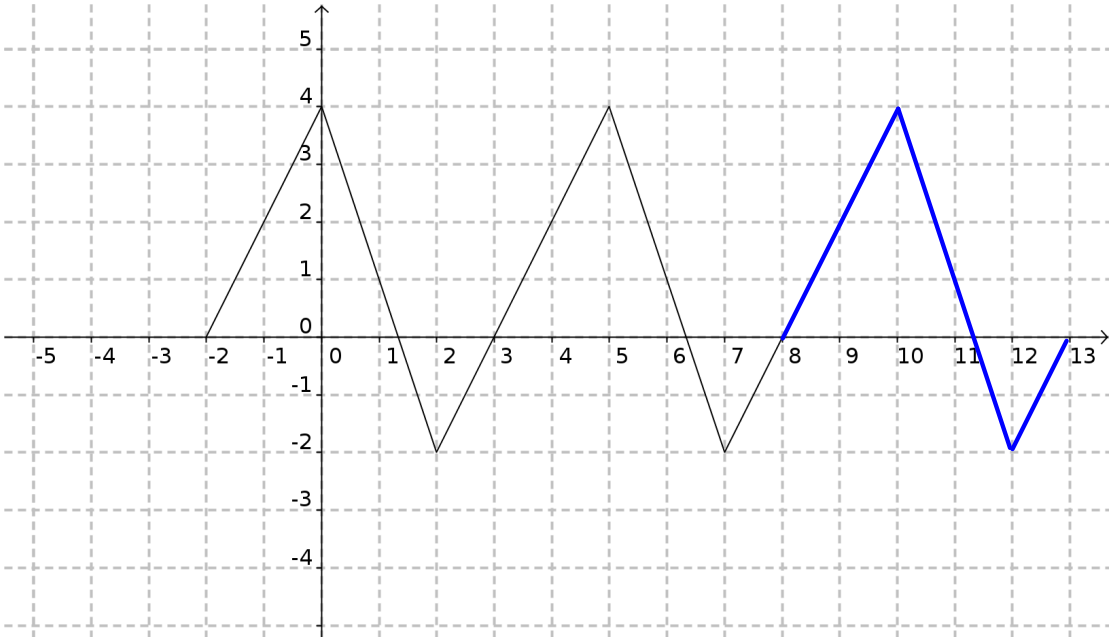
(a) How many cycles are shown in the graph? 2

Ex.1



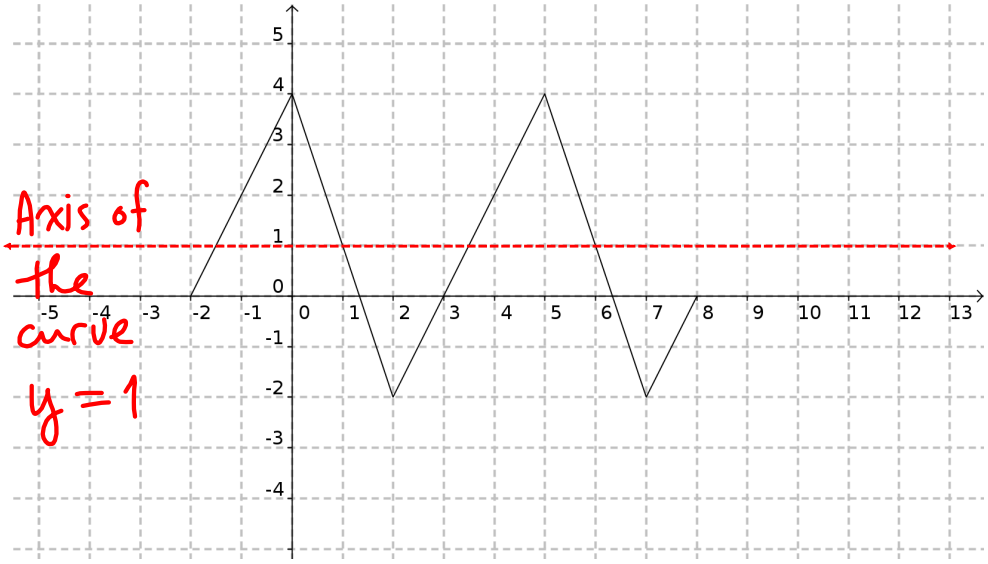
(b) What is the period? 5 units

Ex.1



(c) Draw an additional cycle on the graph.

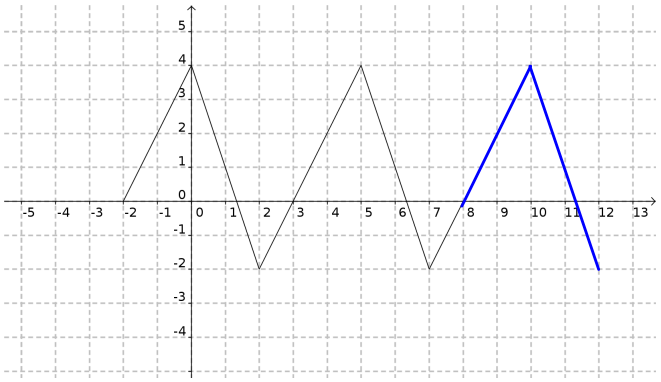
Ex.1



(d) For the given periodic function, state the:

(i) max value 4 (ii) min value -2 (iii) amplitude 3

Ex.1



(e) Suppose the periodic behaviour continues.
What, then, is the value of:

(i) $f(12) = -2$

period = 5

$f(12) = f(12-5)$

$= f(7)$

$= f(2)$

$= -2$

(ii) $f(-4) = f(-4+5)$

$= f(1)$

$= 1$

(e) Suppose the periodic behaviour continues.
What, then, is the value of:

(i) $f(35)$ (ii) $f(71)$

$= f(30)$

$= f(25)$

$\vdots$ brute force

OR

$\frac{35}{\text{period}} = \frac{35}{5}$

$= 7$ ← # of cycles in 35

$f(35) = f(35 - 7(5))$

$= f(0)$

$= 4$

$\frac{71}{\text{period}} = \frac{71}{5}$

$= 14.2$

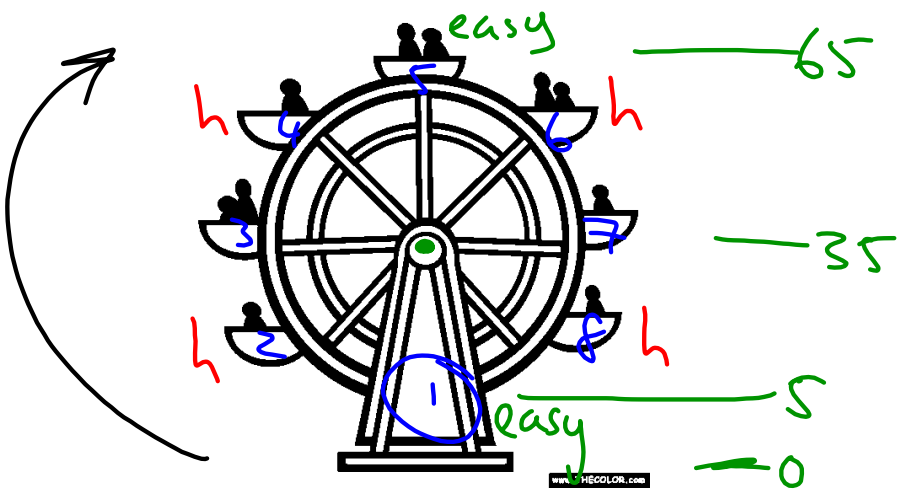
↑ 14 periods in 71

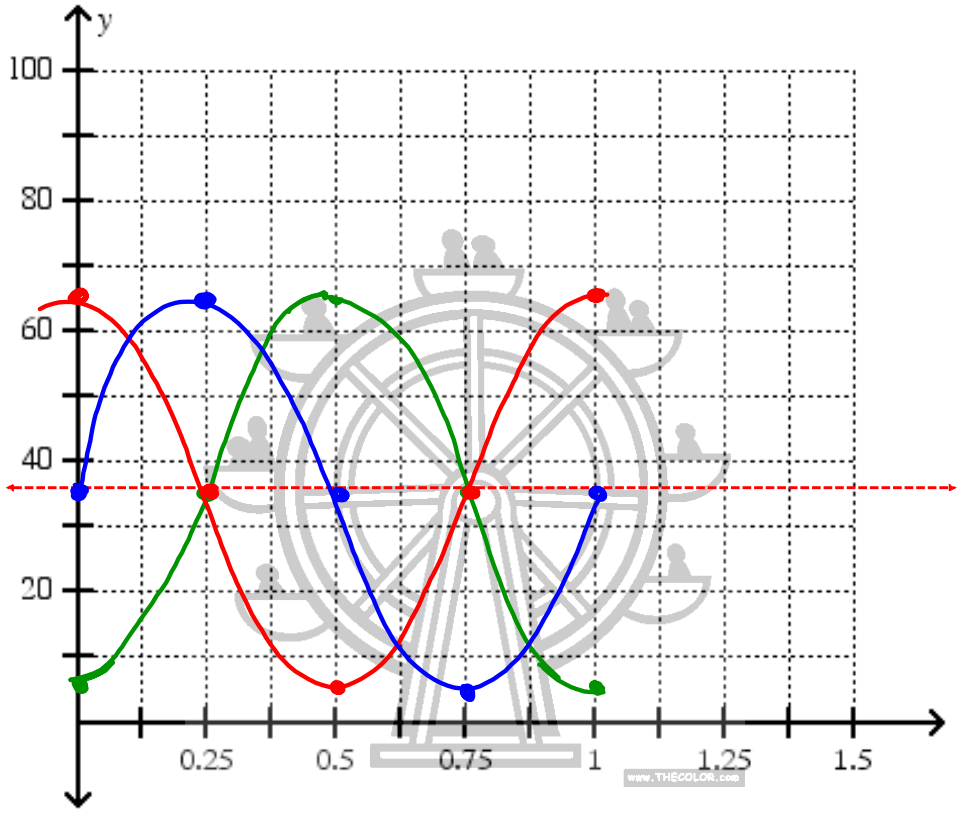
$f(71) = f(71 - 14(5))$

$= f(1)$

Ex.2 A Ferris wheel has a diameter of 60 m and stands 5 m above the ground. As it rotates, the height of a particular seat above the ground is changing. It takes 1 minute for the Ferris wheel to make one complete revolution.

Pick a seat and graph its height as a function of time.



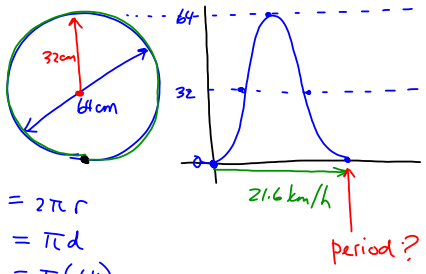


Ex.3 Sketch a graph of a periodic function with a period of 6 and amplitude of 4. Compare your graph with a classmate. Are they the same? Different? Are they each correct?

Assigned Work:

p.352 # 1-9, (11)

11.



$C = 2\pi r$
$= \pi d$
$= \pi(64)$
$\doteq 201.1$ cm

$d_{1\text{ period}} = 201.0619$ cm
$= 2.010619$ m

$v = 21.6$ km/h
$= \frac{21.6\text{ km}}{h}$
$= \frac{21600\text{ m}}{h}$

$d = vt$

$t = \frac{d}{v}$

$= \frac{2.010619\text{ m}}{\frac{21600\text{ m}}{h}}$

$\doteq 0.000093084\text{ h} \times \frac{60\text{ min}}{1\text{ h}} \times \frac{60\text{ s}}{1\text{ min}}$

$\doteq 0.335$ s