Trigonometric Identities

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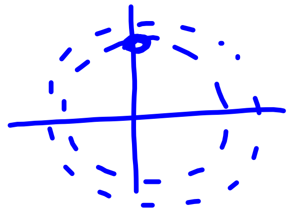
An identity is an equation which is always true for all valid values of the variable.



e.g., $(x+1)^2 = x^2 + 2x + 1$

In the x-y plane, trig ratios are expressed in terms of x, y, and r.

$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$ $\quad$ where: $r^2 = x^2 + y^2$

We will use these definitions to develop some of the fundamental trig identities.

1. Quotient Identity

Consider $\frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}}$

$$= \frac{y}{r} \cdot \frac{r}{x}$$

$$= \frac{y}{x}$$

$$\boxed{\frac{\sin\theta}{\cos\theta} = \tan\theta}$$

Conventions in Trigonometry:

1. Brackets are required around the entire argument of a multi-term argument.

   write sin(x+2), not sinx+2 (looks like sin(x) + 2)
   sin 25° is acceptable, as is cos x, or tan θ.

2. Exponents (other than -1) are written between the function symbol and the argument.

   $(\sin x)^2$ is written as $\sin^2 x$
   $\cos^{-1}\theta$ is the *inverse cosine*
   $(\cos\theta)^{-1}$ is the reciprocal of $\cos\theta$

2. Pythagorean Identity

Consider $\sin^2\theta + \cos^2\theta$

$$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$$

$$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$$

$$= \frac{y^2 + x^2}{r^2}$$

$$= \frac{r^2}{r^2}$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

Tips for working with trig identities:

1. Start with the most complicated side and try to make it simpler.

2. Express tangent in terms of sine and cosine.

3. Look for fundamental identities (quotient, Pythagorean).

4. Only work on one side at a time. Only switch sides if you cannot progress any further.

Ex.1 Prove $\tan\theta\cos\theta = \sin\theta$

$$LS = \tan\theta\cos\theta$$
$$= \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)\frac{\cancel{\cos\theta}}{1}$$
$$= \sin\theta$$
$$= RS \checkmark$$

Ex.2 Prove $\tan^2\theta = \sin^2\theta\cos^{-2}\theta$

$$RS = (\sin^2\theta)\left(\frac{1}{\cos^2\theta}\right)$$
$$= \frac{\sin^2\theta}{\cos^2\theta}$$
$$= \left(\frac{\sin\theta}{\cos\theta}\right)^2$$
$$= (\tan\theta)^2$$
$$= LS \checkmark$$

Ex.3 Prove $\dfrac{\sin^2\theta}{1-\cos\theta} = 1+\cos\theta$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

LS

$$= \frac{\sin^2\theta}{1-\cos\theta}$$
$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$
$$= \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$$
$$= 1+\cos\theta$$

RS

$$= 1+\cos\theta$$

$LS = RS \checkmark$

Assigned Work:

p.310 # 1-3, 4, 5, 6, 7, 8, 11 ,12, 14
+ worksheet

2a d b b

Note: The text asks for restrictions in many questions. Restrictions on trig identities can be quite complex and are generally NOT required or requested. You are proving the identity for all valid values of the variable(s).

2(a) $(1-\sin\alpha)(1+\sin\alpha)$

$= (1-x)(1+x)$

$= 1-x^2$

$= 1-\sin^2\alpha$

$= \cos^2\alpha$

$\sin^2\theta+\cos^2\theta=1$

$\cos^2\theta=1-\sin^2\theta$

5(d) $1-\cos^2\theta = \sin\theta\cos\theta\tan\theta$

$LS = 1-\cos^2\theta$

$= \sin^2\theta$

$= \sin\theta\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right) = \sin^2\theta$

$= \sin\theta\cos\theta\tan\theta$

$RS = \sin\theta\cos\theta\tan\theta$

$= \sin\theta\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)$

$LS = RS$ ✓

7(a) $\sin\theta \cot\theta - \sin\theta\cos\theta$

$= \sin\theta\left(\frac{\cos\theta}{\sin\theta}\right) - \sin\theta\cos\theta$

$= \cos\theta - \sin\theta\cos\theta$

$= \cos\theta(1 - \sin\theta)$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta}{\sin\theta}$

(b) $\cos\theta(1+\sec\theta)(\cos\theta - 1)$

$= \cos\theta\left(1 + \frac{1}{\cos\theta}\right)(\cos\theta - 1)$

$= (\cos\theta + 1)(\cos\theta - 1)$

$(x+1)(x-1)$

$= \cos^2\theta - 1$

$= -\sin^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta - 1 = -\sin^2\theta$

12(b) $\sin^2\alpha - \cos^2\alpha - \tan^2\alpha = \frac{2\sin^2\alpha - 2\sin^4\alpha - 1}{1-\sin^2\alpha}$

$RS = \frac{2\sin^2\alpha - 2\sin^4\alpha - 1}{1-\sin^2\alpha}$

let $x = \sin^2\alpha$

$= \frac{2x - 2x^2 - 1}{1-x}$

$= \frac{-2x^2 + 2x - 1}{1-x}$

$RS = \frac{-[2x^2 - 2x + 1]}{1-x}$

M 2
A −2
N ✗

$LS = \sin^2\alpha - \cos^2\alpha - \tan^2\alpha$

$= \sin^2\alpha - \cos^2\alpha - \frac{\sin^2\alpha}{\cos^2\alpha}$

$= \sin^2\alpha - (1-\sin^2\alpha) - \frac{\sin^2\alpha}{1-\sin^2\alpha}$

$= \frac{(1-\sin^2\alpha)(2\sin^2\alpha - 1)}{1-\sin^2\alpha} - \frac{\sin^2\alpha}{1-\sin^2\alpha}$

$= \frac{(2\sin^2\alpha - 1)(1-\sin^2\alpha) - \sin^2\alpha}{1-\sin^2\alpha}$

$= \frac{2\sin^2\alpha - 2\sin^4\alpha - 1 + \sin^2\alpha - \sin^2\alpha}{1-\sin^2\alpha}$

$= \frac{2\sin^2\alpha - 2\sin^4\alpha - 1}{1-\sin^2\alpha}$

$= RS$ ✓

$\frac{c \cdot a}{c} + \frac{b}{c} = $ ______