Unit 7: Discrete Functions
Patterns and Sequences

Dec 19/2019

Definitions:

1) A sequence is an ordered list of terms (numbers and/or variables).

$\in$, "member of"

The terms are denoted by t_n, $n \varepsilon \mathbb{N}$.

natural numbers, 1, 2, 3, 4, ...

t_1 t_2 t_3 t_4

ex: 1, 3, 5, 7, ...

note: 3 is t_2, since it is the second term in the sequence.

$\mathbb{R}$ $\mathbb{N}$ $\mathbb{Z}$ $\mathbb{I}$ $\mathbb{W}$ $\mathbb{C}$

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note: 3 is t_2, since it is the second term in the sequence.

2) A finite sequence is a sequence with a specific number of terms (i.e., the list ends).

ex: 1, 3, 5, 7, 9. (*there are 5 terms*)

3) An infinite sequence is a sequence that continues without end (i.e., there are infinite terms).

ex: 1, 3, 5, 7, 9,11, ...

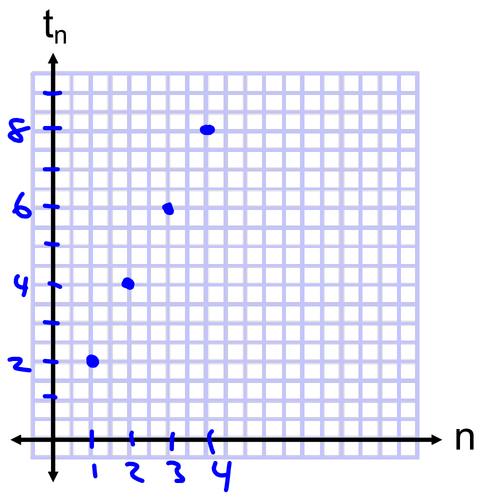
Note:

1) The number of terms in a sequence is a natural number ($n \in \mathbf{N}$), thus we say it is discrete.

2) A sequence can be plotted on a grid, but the points are not joined since each term is discrete.

ex: 2, 4, 6, 8

n	t_n
1	$t_1 = 2$
2	$t_2 = 4$
3	$t_3 = 6$
4	$t_4 = 8$



Sequences that have a pattern can be described using an algebraic expression.

(a) The general term, t_n, can be used to find any term.

(b) The recursive definition uses information about the previous term(s).

(c) A discrete function is the same as the general term, written in function notation.

Ex. Find the pattern that relates the term value with the term number for 3, 6, 9, 12, ...

$t_1 = 3$
$t_2 = 6$
$t_3 = 9$

Ex. Find the pattern that relates the term value with the term number for 6, 9, 12, ...

3

$t_1 = 3$
$t_2 = 6$
$t_3 = 9$

(a) $t_n = 3n$ $\qquad t_{100} = 3(100) = 300$

(b) $t_n = t_{n-1} + 3$ where $t_1 = 3$ $\qquad t_{100} = t_{99} + 3$

(c) $f(n) = 3n$

Ex. Determine the general term and recursive definition.

a) 10, 15, 20, ...

$t_n = t_{n-1} + 5$ where $t_1 = 10$

$t_n = 5n + 5 = 5(n+1)$

b) 1, 4, 9, 16, ...

$t_n = n^2$

$t_n = \left(\sqrt{t_{n-1}} + 1\right)^2$ ✓

$t_5 = \left(\sqrt{t_4} + 1\right)^2 = \left(\sqrt{16} + 1\right)^2 = (4+1)^2 = 25$ ✓

where $t_1 = 1$

y	Δy	$\Delta^2 y$
1		
	3	
4		2
	5	
9		2
	7	
16		

quadratic

1, 4, 9, 16, 25

+3 +5 +7

$t_2 = t_1 + 3$
$t_3 = t_2 + 5$
$t_4 = t_3 + 7$

$t_n = t_{n-1} + 2n - 1$ where $t_1 = 1$

Ex 2) Find the first three terms in each of the following sequences

a) $t_n = 2n + 1$

$t_1 = 2(1) + 1$
$= 3$

$t_2 = 2(2) + 1$
$= 5$

$t_3 = 2(3) + 1$
$= 7$

b) $t_n = n^2 + 4$

$t_1 = (1)^2 + 4$
$= 5$

$t_2 = (2)^2 + 4$
$= 8$

$t_3 = (3)^2 + 4$
$= 13$

Assigned Work:

worksheet

4, 16

4. $\frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \frac{5}{32}, \ldots$

$\frac{16}{32}, \frac{16}{32}, \frac{12}{32}, \frac{8}{32}, \frac{5}{32}, \frac{30}{320} = \frac{3}{32}$

$\frac{2}{2}$

$\times 1$, $\times \frac{3}{4}$, $\times \frac{2}{3} = \frac{4}{6}$, $\times \frac{5}{8}$, $\times \frac{6}{10}$, $\times \frac{7}{12}$, $\times \frac{8}{14}$

-0, $-\frac{4}{32}$, $-\frac{4}{32}$, $-\frac{3}{32}$

16. $37, 46, 55, 64, 73, \ldots$

$+9$, $+9$

$37 + 9(9) = 37 + 81$

17. $a_n = \frac{2n+1}{n^3}$

$a_1 = \frac{2(1)+1}{(1)^3}$
$= 3$

$a_2 = \frac{2(2)+1}{(2)^3}$
$= \frac{5}{8}$

$a_3 = \frac{2(3)+1}{3^3}$
$= \frac{7}{27}$

$a_4 = \frac{2(4)+1}{4^3}$
$= \frac{9}{64}$

$\frac{3}{1}, \frac{5}{8}, \frac{7}{27}, \frac{9}{64}, \frac{11}{125}$