Unit 7: Discrete Functions
Arithmetic & Geometric Sequences

Jan 6/2020

Ex.1 Find the next 3 terms in each sequence:

(a) 3, 7, 11, 15, ... , 19, 23, 27

+4 +4

(b) 9, 4, -1, -6, ... , -11, -16, -21

-5

(c) 1, 1.25, 1.5, 1.75, ..., 2, 2.25, 2.5

+0.25

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Ex.2 For the sequence 3, 9, 15, 21, ...

+6 +6 +6

(a) find the 100th term.
(b) find a general expression for the nth term.

(a) $t_{100} = 3 + 99(6)$

start

(b) $t_n = 3 + (n-1)(6)$

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An arithmetic sequence is a *linear function* where the difference between consecutive terms is a constant (called the common difference, *d*).

The first term, t_1, or $f(1)$, is a.

In general, the sequence is:

$$a,\ a+d,\ a+2d,\ a+3d,\ \ldots$$

The n^{th} term is:

$$t_n = a + (n-1)d \quad \text{or} \quad f(n) = a + (n-1)d$$

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Ex.3 How many terms are in the finite sequence

16, 7, -2, -11, ..., -245?

$d = -9$

$a = 16 = t_1$

$t_n = a + (n-1)d$

$t_n = 16 + (n-1)(-9)$

set $t_n = -245$

$-245 = 16 + (n-1)(-9)$

$-261 = (n-1)(-9)$

$\frac{-261}{-9} = n-1$ $\quad\therefore$ 30 terms.

$29 = n-1$

$n = 30$

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Ex.4 Find the next three terms in each sequence:

(a) 2, 4, 8, 16, ..., 32, 64, 128

$\times 2$

(b) 1, -2, 4, -8, ..., 16, −32, 64

$\times(-2)$

$t_n = (1)(-2)^{n-1}$

(c) 27, 9, 3, 1, ..., $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$

$\div 3$

$\times \frac{1}{3}$

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Ex.5 For the sequence 5, 10, 20, 40, ..., 80, 160, 320, 640

$\times 2$

(a) find the 8th term.
(b) write an expression for the nth term.
(c) where would you find 5120 in the sequence?

(a) $t_8 = (5)(2)^7$

start

$= 5(128)$

$= 640$

(b) $t_n = (5)(2)^{n-1}$ $t_n = ar^{n-1}$

(c) $t_n = 5120$

$\frac{5(2)^{n-1}}{5} = \frac{5120}{5}$

$2^{n-1} = 1024$

$2^{n-1} = 2^{10}$

$\Rightarrow n-1 = 10$

$n = 11$

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A geometric sequence occurs when there is a common ratio (r) between consecutive terms.

The first term, t_1, or $f(1)$, is a.

In general, the sequence is:

$$a,\ ar,\ ar^2,\ ar^3,\ \ldots$$

The n[th] term is:

$$t_n = ar^{n-1} \quad \text{or} \quad f(n) = ar^{n-1}$$

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Ex.6 Is each sequence geometric? If so, state the common ratio.

(a) $2, -8, 32, -128, \ldots$

(b) $x, 2x, 3x, 4x, \ldots$

(c) $x^7, x^{14}, x^{28}, x^{56}, \ldots$

(d) $2x^7, 4x^{10}, 8x^{13}, 16x^{16}, \ldots$

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Ex.7 Given t_5 = 1875 and t_7 = 46875, find t_n (geometric).

$t_n = ar^{n-1}$

$1875 = ar^4$ ① $46875 = ar^6$ ②

② ÷ ① $\frac{46875}{1875} = \frac{ar^6}{ar^4}$

$25 = r^2$

$r = \pm 5$

Sub $r = 5$ in ①
$1875 = a(5)^4$
$1875 = 625a$
$a = 3$

sub $r = -5$ in ①
$1875 = a(-5)^4$
$1875 = 625a$
$a = 3$

$t_n = 3(5)^{n-1}$ OR $t_n = 3(-5)^{n-1}$

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Assigned Work:

p.424 # 5, 6, 8odd e, 10, 12, 13odd, 15
p.430 # 6odd, 7, 8, 11, 14

8(e) $1, \frac{6}{5}, \frac{7}{5}, \ldots$ $t_3 - t_2 =$ ___

$= \frac{5}{5}, \frac{6}{5}, \frac{7}{5}, \ldots$ $t_2 - t_1 =$ ___

$+\frac{1}{5}$ $\frac{1}{5}$

$d = \frac{1}{5}$

$a = 1$

$t_n = 1 + (n-1)\left(\frac{1}{5}\right)$

$= 1 + \frac{1}{5}n - \frac{1}{5}$

$= \frac{4}{5} + \frac{1}{5}n$

$= \frac{1}{5}n + \frac{4}{5}$

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12 $P = \$5000$ $I = 3.5\%$

simple interest → applies to \$5000 only each year.

$t_1 = 5000$

$t_2 = 5000 + \underbrace{(0.035)(5000)}_{\text{interest in one year}}$

175

$t_3 = 5000 + 2(175)$ $d = 175$

$t_4 = 5000 + 3(175)$ $a = 5000$

$t_n = 5000 + (n-1)(175)$

$7800 = 5000 + (n-1)(175)$

$\frac{2800}{175} = \frac{(n-1)(175)}{175}$

$16 = n - 1$

$\underbrace{n = 17}$

only represents 16 years of interest

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15. $t_{50} = 238$ $t_{93} = 539$

$a + (49)d = 238$ ① $a + 92d = 539$ ②

$a + 49d = 238$ ①

② − ① $43d = 301$

$d = 7$

Sub $d = 7$ into ①

$a + 49(7) = 238$

$a = -105$

$t_n = -105 + (n-1)(7)$

$= -105 + 7n - 7$

$= -112 + 7n$

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11. $t_5 = 45$ $t_8 = 360$

$$t_n = ar^{n-1}$$

$ar^4 = 45$ ① $ar^7 = 360$ ②

$ar^4 = 45$ ①

② ÷ ① $r^3 = 8$

$\boxed{r = 2}$

OR $a = \frac{45}{r^4}$ ③

sub ③ into ② $ar^7 = 360$

$\left(\frac{45}{r^4}\right) r^7 = 360$

$ar^4 = 45$

$a(2)^4 = 45$ $t_n = \frac{45}{16}(2)^{n-1}$

$a = \frac{45}{16}$ $t_{20} = \frac{45}{16}(2)^{19}$

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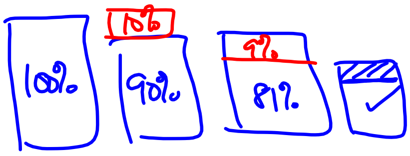
14. reduce by 10% → 90% remains

start with N bacteria

#doses	
0	$t_1 = N$
1	$t_2 = N(0.9)$
2	$t_3 = N(0.9)^2$
3	$t_4 = N(0.9)^3$
4	$t_5 = N\underbrace{(0.9)^4}$



$0.6561 = 65.61\%$

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