Recursion Formulae & Recursive Sequences

Determine the pattern in the Fibonacci sequence:

1, 1, 2, 3, 5, 8, ... $n \; \varepsilon \; \mathbb{N}$

, 13, 21, 34, . . .

$t_1 = 1$

$t_2 = 1$

$t_3 = t_2 + t_1$

$t_4 = t_3 + t_2$

$t_n = t_{n-1} + t_{n-2}$

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Recursion & Recursive Sequences

Jan 7/2020

A sequence is recursive if a new term is found using one or more previous terms. The starting term(s) must be provided as part of the recursive definition.

For example, the Fibonacci sequence is:

$$t_n = t_{n-1} + t_{n-2}, \text{ where } t_1 = 1 \text{ and } t_2 = 1, \; n > 2$$

Ex. Find the first four terms in each of the following sequences.

a) $t_n = t_{n-1} - 2$, where $t_1 = 3$

$t_1 = 3$ $\quad t_2 = 3 - 2 = 1$ $\quad t_3 = 1 - 2 = -1$ $\quad t_4 = -1 - 2 = -3$

b) f(n) = f(n - 1) + 1.5, where f(1) = -0.5

$f(1) = -0.5$ $\quad f(2) = -0.5 + 1.5 = 1$

$f(3) = 1 + 1.5 = 2.5$ $\quad f(4) = 2.5 + 1.5 = 4$

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Ex. Determine a recursion formula for each of the following sequences, then write an explicit formula if possible (an explicit formula does not rely on recursion).

a) -3, 6, -12, 24, ...

recursive
$t_n = -2t_{n-1}, t_1 = -3$

OR

$t_n = -3(-2)^{n-1}$ explicit/general

b) f(1) = 2, f(2) = 6, f(3) = 10, f(4) = 14, ...

$2, 6, 10, 14, \ldots$ +4 +4

$t_n = t_{n-1} + 4, t_1 = 2$

OR

$t_n = 2 + (n-1)(4)$
$= -2 + 4n$

c) 3, 5, 8, 12, ...

+2 +3 +4 not arithmetic

$\frac{5}{3} \neq \frac{8}{5} \neq \frac{12}{8}$ not geometric

$t_1 = 3$
$t_2 = 5 = 3+2 = 3+n$
$t_3 = 8 = 5+3 = 5+n$
$t_4 = 12 = 8+4 = 8+n$

$t_n = t_{n-1} + n, t_1 = 3$

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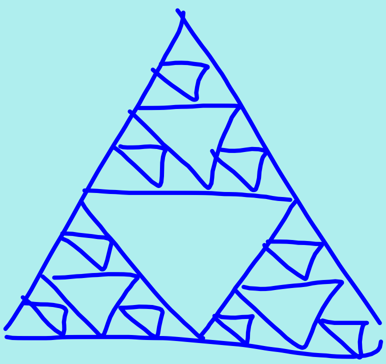
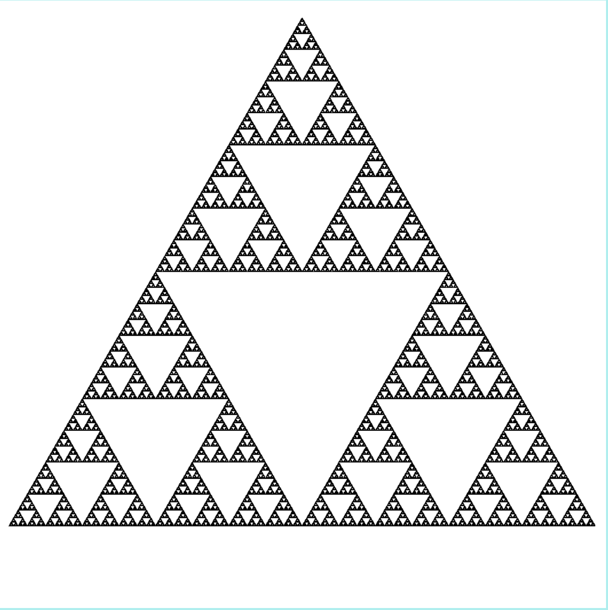
Recursion can also be applied to geometric figures to produce **fractals**, patterns that repeat themselves at any scale (zooming in or out).

One of the simplest and most famous is the Sierpinski Triangle:





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Pascal's Triangle is recursively defined, where each term in any row is the sum of the terms diagonally above that term. The first row is 1.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

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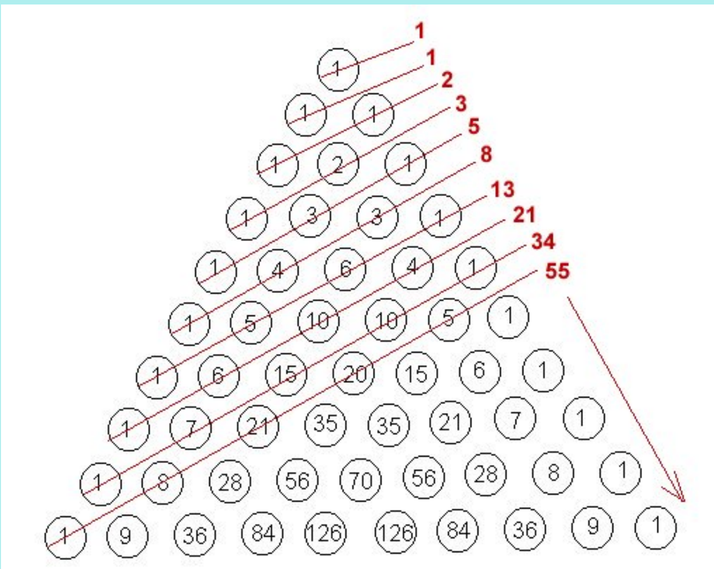
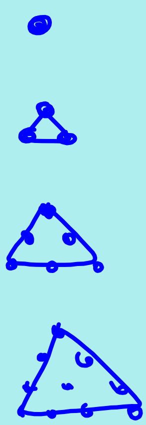
Fibonacci Numbers from Pascal's Triangle

The Fibonacci numbers can be obtained by adding the *shallow diagonals* in Pascal's triangle.





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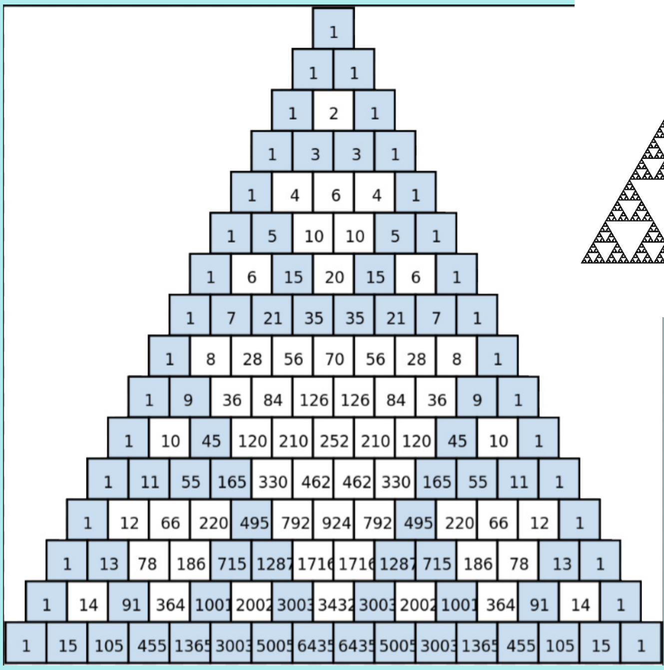
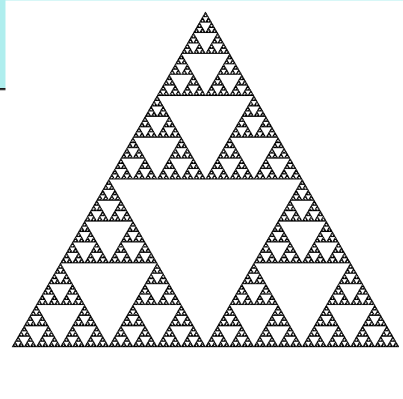
Sierpinski's Triangle from Pascal's Triangle

https://youtu.be/wcxmdiuYjhk





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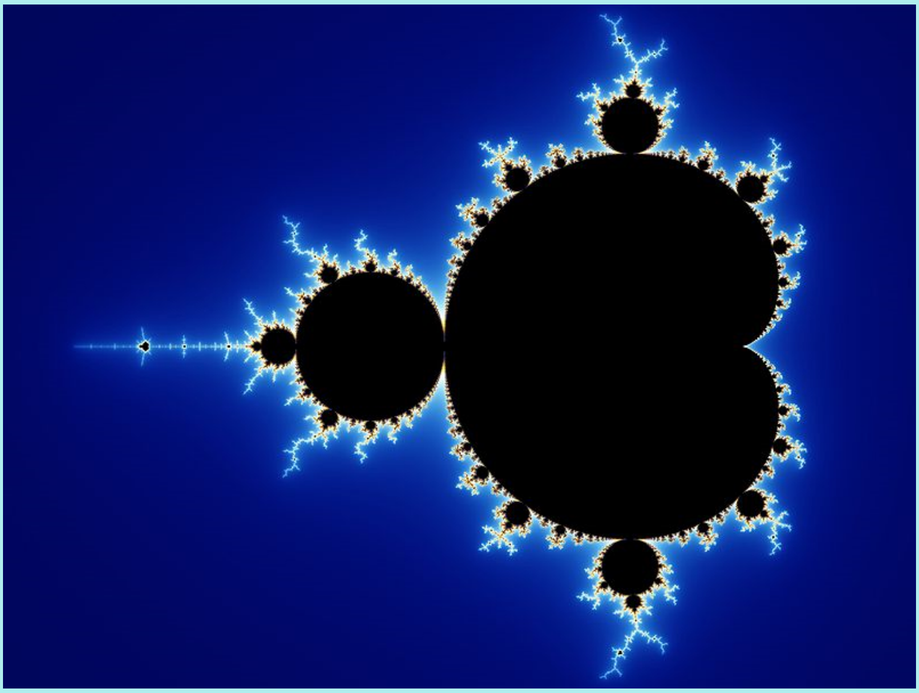
The Mandelbrot Set: A recursively defined set of points



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Nature by Numbers:

https://youtu.be/kkGeOWYOFoA

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