Arithmetic Series

Jan 8/2020

Definitions:
1) A series is the **sum** of the terms in a sequence. The sum is denoted S.
e.g., sequence: $t_1, t_2, t_3, ..., t_n$
series: $t_1 + t_2 + t_3 + ... + t_n = S_n$, where $n \in \mathbb{N}$

note: $S_1 = t_1$
$S_2 = t_1 + t_2$
$S_3 = t_1 + t_2 + t_3$,
etc.

To get from one sum to the next you add the next term, so

$S_n - S_{n-1} = t_n$

nth term

$t_n \rightarrow$ "nth term" OR "term n"

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Definitions:
2) An arithmetic series is the sum of the terms in an arithmetic sequence.

The sum can be found using one of the following formulas:

$$s_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad \text{or} \quad s_n = \frac{n}{2}\left[t_1 + t_n\right]$$

# of terms   1st   last

Recall: $t_n = a + (n - 1)d$

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Ex. Determine S_{30} for the arithmetic series

$5 + 9 + 13 + 17 + ... + (4n+1)$

$t_1 \quad t_2 \quad t_3 \quad t_4$

$t_n = 4n+1$

$S_n = \frac{n}{2}[2a+(n-1)d]$ or $S_n = \frac{n}{2}[t_1+t_n]$

? ?

$9-5=4$

$S_{30} = \frac{30}{2}[2(5)+(29)(4)]$

$= 15[10+116]$

$= 1890$

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Ex. Find the sum of the first 10 terms of the arithmetic series 21 + 16 + 11 + ...

$a=21$

$d=16-21$
$=-5$

$n=10$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{10} = \frac{10}{2}[2(21)+(9)(-5)]$

$= 5[42-45]$

$= 5(-3)$

$S_{10} = -15$

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Ex. Given -25 - 22 - 19 - ... + 32
(a) determine the sum of the series.
(b) determine the sum of the last 5 terms.

(a) $a = -25 = t_1$
$d = 3$
$n = ?$

$t_n = a + (n-1)d$
$32 = -25 + (n-1)(3)$
$57 = (n-1)(3)$
$19 = n-1$
$n = 20$

$S_n = \frac{n}{2}[t_1 + t_n]$
$S_{20} = \frac{20}{2}[-25+32]$
$= 10[7]$
$= 70$

(b) Sum of last 5 terms
$S_{20} - S_{15}$
$$= 70 - \underbrace{\left[\frac{15}{2}\left(2(-25)+(14)(3)\right)\right]}_{S_{15}}$$

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Deriving the formula for arithmetic series:

$$S_n = t_1 + t_2 + t_3 + \ldots + t_{n-2} + t_{n-1} + t_n$$

Recall: $t_n = a + (n - 1)d$

$$S_n = t_1 + (t_1 + d) + (t_1 + 2d) + \ldots + (t_n - 2d) + (t_n - d) + t_n \quad ①$$

Make a 2nd equation by reversing the order of the terms

$$S_n = t_n + (t_n - d) + (t_n - 2d) + \ldots + (t_1 + 2d) + (t_1 + d) + t_1 \quad ②$$

Add equation (1) and equation (2) to eliminate 'd'.

$$2S_n = [t_1 + t_n] + [t_1 + t_n] + [t_1 + t_n] + \ldots + [t_1 + t_n] + [t_1 + t_n] + [t_1 + t_n]$$

There are n of these $[t_1 + t_n]$ terms

$$2S_n = n[t_1 + t_n]$$

$$S_n = \frac{n}{2}[t_1 + t_n]$$ This is one formula

Or, substitute $t_n = a + (n - 1)d$

$$S_n = \frac{n}{2}[a + a + (n-1)d]$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

This is the other formula

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Assigned Work:

p.452 # 1-3, (4-7)(even), 10-12, 15

1d
5f
6f

b, d, . . .

12
10 (start)

x_1 y_1
x_2 y_2
x_3 y_3

1(d) $-78 - 56 - 34 - \dots$

$a = -78$ $\quad d = -56 - (-78)$
$= 22$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{10} = \frac{10}{2}\left[2(-78) + 9(22)\right]$$
$$= 5\left[-156 + 198\right]$$
$$= 210$$

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5(f) $p + (2p + 2q) + (3p + 4q) + \dots$

$$a = p$$
$$d = (2p + 2q) - p$$
$$= p + 2q$$

$$t_n = a + (n-1)d$$
$$t_{12} = p + (11)(p + 2q)$$
$$= p + 11p + 22q$$
$$= 12p + 22q$$

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6(f) $t_7 = 43$ $t_{13} = 109$ $S_{20} = ?$

$t_n = a + (n-1)d$

$a + 6d = 43$ ① $a + 12d = 109$ ②

$a + 6d = 43$ ①

② − ① $6d = 66$

$d = 11$ ✓

$a + 6(11) = 43$

$a = 43 - 66$

$a = -23$ ✓

$n = 20$

S_{20}

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10. $t=0$, 1, 2

$4.9 + (4.9 + 9.8) + (4.9 + 2(9.8)) + \dots$

$+9.8$ $+9.8$

$a = 4.9$ $d = 9.8$

$S_{15} =$

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1 20

12. 3m 20sec 1m 45sec

200 sec 105 sec

$t_1 = 200$ $t_{20} = 105$

$$S_n = \frac{n}{2}\left[t_1 + t_n\right]$$

$$S_{20} = \frac{20}{2}\left[200 + 105\right]$$

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15. $t_{10} = 34$ $S_{20} = 710$

$t_n = a + (n-1)d$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$a + 9d = 34$ (1) $\frac{20}{2}\left[2a + 19d\right] = 710$ (2)

OR

$a = 34 - 9d$ $10\left[2a + 19d\right] = 710$

$20a + 38d = 710$ (3)

(1) $\times 20$: $20a + 180d = 680$ (4)

Sub into (3)

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