Geometric Series

Jan 9/2020

A geometric series is the sum of the terms in a geometric sequence.

The sum can be found using the following formula:

$$s_n = \frac{a\left(r^n - 1\right)}{r-1}, r \neq 1$$

Recall: $t_n = ar^{n-1}$

Ex. For the geometric series with a = 7 and r = 2, determine S_9.

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$S_9 = \frac{7(2^9 - 1)}{2-1}$$

$$= \frac{7(512-1)}{1}$$

$$= 7(511)$$

$$= 3577$$

Ex. Determine the sum of the series

15 + 45 + 135 + 405 + ... + 32 805

$a = 15$

$r = \frac{45}{15}$ or $\frac{135}{45}$

$r = 3$

$n = ?$

$t_n = ar^{n-1}$

$\frac{32805}{15} = \frac{15(3)^{n-1}}{15}$

$2187 = 3^{n-1}$

$3^7 = 3^{n-1}$

$\Rightarrow 7 = n-1$

$n = 8$

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_8 = \frac{15(3^8 - 1)}{3-1}$

$= \frac{15(6560)}{2}$

$S_8 = 49200$

Ex. Find the common ratio, r, for the geometric series with a=7 and S_3=217.

$7, 7r, 7r^2$

$a = 7$

$r = ?$

$n = 3$

$S_3 = 217$

$S_n = \frac{a(r^n - 1)}{r-1}$

$217 = \frac{7(r^3 - 1)}{r-1}$

$S_3 = 7 + 7r + 7r^2$

$\frac{217}{7} = \frac{7 + 7r + 7r^2}{7}$

$31 = 1 + r + r^2$

$0 = r^2 + r - 30$

$0 = (r+6)(r-5)$

$r = -6$ or $r = 5$

$7, -42, 252, \ldots$ $7, 35, 175, \ldots$

Ex. Aleah is doing a series of experiments involving a chemical. In the first experiment, she uses 10 g of the chemical, and in each subsequent experiment, she increases the amount of the chemical used by 25%. If she has 130 g of chemical available to her, how many experiments can she do?

$a = 10$

$r = 100\% + 25\%$
$= 125\%$
$= 1.25$

$n = ?$

$S_n = 130$

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$130 = \frac{10(1.25^n - 1)}{1.25 - 1}$$

$$0.25 \times 13 = \frac{1.25^n - 1}{0.25} \times 0.25$$

$$4.25 = 1.25^n$$

try $n = 10$

$1.25^{10} \doteq 9.3$

$1.25^5 \doteq 3.1$

$1.25^6 \doteq 3.8$

too far $\longrightarrow 1.25^7 \doteq 4.76$

$n = 6$ $\therefore$ she can perform 6 experiments.

Deriving the formula for geometric series:

$$S_n = t_1 + t_2 + t_3 + \ldots + t_{n-2} + t_{n-1} + t_n$$

Recall: $t_n = ar^{n-1}$, thus

$$S_n = a + ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1} \quad (1)$$

Multiply equation 1 by *r*

$$rS_n = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n \quad (2)$$

Subtract the two equations so that most of the terms eliminate.

$$S_n - rS_n = a - ar^n$$

Factor S_n on the left and factor *a* on the right side.

$$S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{(1-r)}, \; r \neq 1$$

To obtain the same equation as in our notes multiply both numerator and denominator by -1.

$$S_n = \frac{a(r^n-1)}{(r-1)}, \; r \neq 1$$ This is the formula

Assigned Work:

p.459 # 2, (3-6)(even), 7, 10, 12, 15, 16

3d 5bf
6f

5(b) $a = 11 = t_1 \quad t_7 = 704$

$$S_7 = ?$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$t_n = ar^{n-1}$

$704 = 11r^6$

$64 = r^6$

$r = \pm\sqrt[6]{64}$

$r = \pm 2$

$r = 2: \; S_7 = \frac{11(2^7 - 1)}{2 - 1}$

$= \frac{11(127)}{1}$

$= 1397$

$r = -2: \; S_7 = \frac{11((-2)^7 - 1)}{-2 - 1}$

$= \frac{11(-129)}{-3}$

$= 473$

3(d) $\frac{4}{5} + \frac{8}{15} + \frac{16}{45} + \cdots \qquad t_6 \quad S_6$

$r = \frac{2}{3}$ ✓

$a = \frac{4}{5} \qquad t_n = ar^{n-1}$

$r = \frac{2}{3} \qquad t_6 = \left(\frac{4}{5}\right)\left(\frac{2}{3}\right)^5$

$= \left(\frac{4}{5}\right)\left(\frac{32}{243}\right)$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_6 = \frac{\left(\frac{4}{5}\right)\left(\left(\frac{2}{3}\right)^6 - 1\right)}{\frac{2}{3} - 1}$$

$$= \frac{\frac{4}{5}\left(\frac{64}{729} - 1\right)}{-\frac{1}{3}} \qquad 1 = \frac{729}{729}$$

$$= -\frac{12}{\cancel{5}_1}\left(-\frac{\cancel{665}^{133}}{729}\right)$$

$$= \frac{1596}{729}$$

5 (f) $t_5 = 5 \quad t_8 = -40$

$① \; 5 = ar^4 \qquad -40 = ar^7 \; ②$

$$5 = ar^4$$

$② \div ① \qquad -8 = r^3$

$$r = -2$$

6 (f) $4 + 2 + 1 + \dots + \frac{1}{1024}$

$t_n = ar^{n-1}$

$a = 4$

$r = \frac{1}{2}$

$n = ?$

$$\frac{\frac{1}{1024}}{4} = \frac{4\left(\frac{1}{2}\right)^{n-1}}{4}$$

$$\frac{1}{4096} = \left(\frac{1}{2}\right)^{n-1}$$

$$\left(\frac{1}{2}\right)^{12} = \left(\frac{1}{2}\right)^{n-1}$$

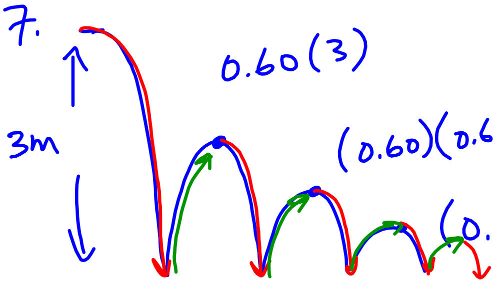
OR

$4096 = 2^{n-1} \quad \Rightarrow \quad 12 = n - 1$

$$n = 13$$

$$\left(\frac{3}{2}\right)^{n-1} = \frac{243}{32}$$

7.



$0.60(3)$

$(0.60)(0.60)(3) = (0.60)^2(3)$

$(0.60)^3(3)$

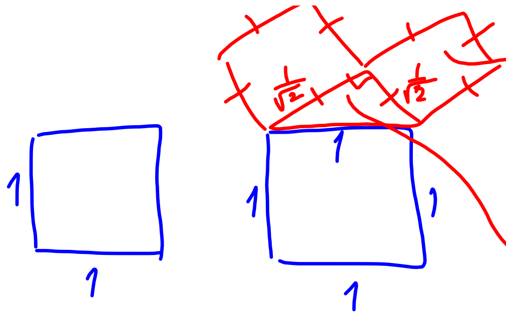
$t_1 = 3$ $\quad a = 3$

$t_2 = 3(0.60)$ $\quad r = 0.60$

$t_3 = 3(0.60)^2$ $\quad S_n = \dfrac{a(r^n - 1)}{r-1}$

$$d_T = d_{down} + d_{up}$$
$$= \frac{3(0.60^5 - 1)}{0.6 - 1} + \frac{0.6(3)(0.6^4 - 1)}{0.6 - 1}$$

10.



$A = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$
$= \frac{1}{2}$

$A = \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$
$= \frac{1}{4}$

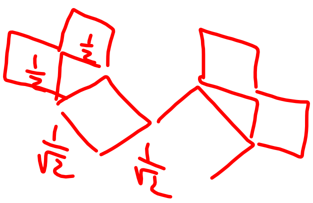
$A_1 = 1$ $\quad A_2 = 1 + \left(\frac{1}{4} + \frac{2}{2}\right)$

$= \frac{9}{4}$

$= 2.25$

$a = 1$

$d = \frac{5}{4}$



$A_3 = 1 + \frac{5}{4} + \frac{5}{4}$