Annuities - Present Value

Jan 16/2020

An annuity is an investment where equal payments (or withdrawls) are made at regular intervals. In an ordinary annuity the payments/withdrawls are made at the end of each interval.

The future value of an annuity is the amount of money you will have (saved up): Your deposits plus the interest each of them has earned.

The present value of an annuity represents the amount of money needed to invest **today** in order to provide regular payments/withdrawls over a future period of time.

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Suppose you wanted to have $1000 available to withdraw, 3 years from now. How much should you invest at 3%/a, compounded annually?

$$PV = \frac{FV}{(1+i)^n} \quad \text{or} \quad PV = FV(1+i)^{-n}$$

$$\begin{aligned} PV &= 1000(1+0.03)^{-3} \\ &= 915.14 \end{aligned}$$

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What about two years? $PV = 1000(1+0.03)^{-2}$

One year? $PV = 1000(1+0.03)^{-1}$

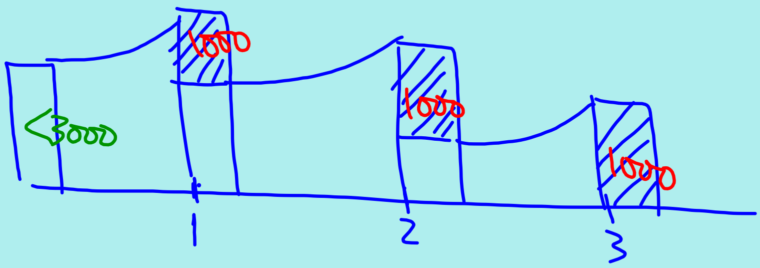
What is you wanted to withdraw this amount each year for three years? How much would you have to invest now?

$$1000(1+0.03)^{-1} + 1000(1+0.03)^{-2} + 1000(1+0.03)^{-3}$$

will grow for one year and then be withdrawn

will grow for two years and then be withdrawn

will grow for three years and then be withdrawn



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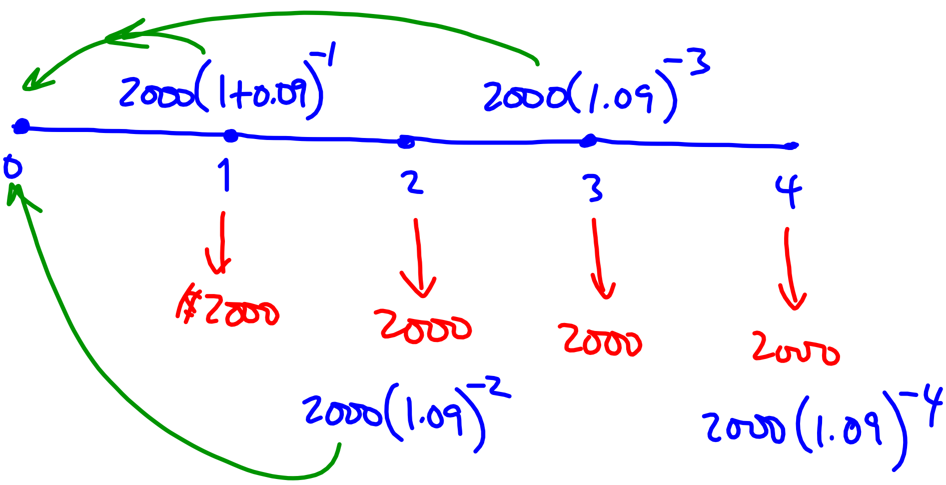
Ex. Consider an annuity which provides 9%/a interest compounded annually and provides annual withdrawals of $2000 for four years (after which it is depleted).

a) Draw a time line



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Ex. Consider an annuity which yields 9%/a interest compounded annually, and provides annual withdrawals of $2000 for four years.

b) Calculate the value of investments required for each withdrawal.

$2000(1.09)^{-1} \doteq 1834.86$ $\quad 2000(1.09)^{-3} = 1544.37$

$2000(1.09)^{-2} = 1683.36$ $\quad 2000(1.09)^{-4} = 1416.85$

c) Determine the initial value of the annuity.

~~\$~~ 6479.44

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How does the work in the time line and the value of each payment relate to sequences and series?

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Present value formula:

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

Ex.4 Determine the present value required for \$75 quarterly withdrawals for 10 years at 9.6%/a compounded $\underbrace{\text{quarterly}}_{\text{4 times per year}}$.

$R = 75$

$n = 10(4)$
$= 40$

$i = \frac{0.096}{4}$

$$PV = \frac{75\left[1-\left(1+\frac{0.096}{4}\right)^{-40}\right]}{\frac{0.096}{4}}$$

$\doteq 1914.82$

$\therefore$ invest \$1914.

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Assigned Work: Annuities - Present Value

p.520 # 3ac, 4, 9, 14

4. $i = \frac{0.18}{12}$

$PV = 1300$

$n = 2(12)$
$= 24$

$R = ?$

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$FV = \frac{R\left[(1+i)^{n}-1\right]}{i}$$

$$R = \frac{PV \cdot i}{1-(1+i)^{-n}}$$

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9.

$PV = 32000$
$2.4\%/a$
monthly
$n = 60$
$i = \frac{0.024}{12}$
$R = ?$

$PV = 29000$
$5.4\%/a$
monthly
$n = 60$
$i = \frac{0.054}{12}$
$R = ?$

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14.

2020 → 2045
Savings

$R = ?$

$i = \frac{0.09}{12}$

$n = 25(12)$
$= 300$

$FV = \bigcirc$

Same as savings in 2045

$FV = \frac{R[(1+i)^n - 1]}{i}$

$\therefore$

$R = \frac{\quad}{\quad}$

2045 → 2060
retires

$R = 2500$

$n = 15(12)$
$= 180$

PV 15

① $PV = \frac{R[1-(1+i)^{-n}]}{i}$

$\therefore$

$\underbrace{PV}_{\text{amount at start of retirement 2045}} = \frac{\quad}{\quad}$

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