Combinations

Feb 25/2020

Permutation: An arrangement of identifiable elements where order matters.

$$ {}_nP_r = P(n,r) = \frac{n!}{(n-r)!} $$

"n permutations of r elements"

Combination: An arrangement of elements where order does not matter.

$$ {}_nC_r = C(n,r) = \frac{n!}{(n-r)!r!} $$

"n combinations of r elements"
or
"n choose r"

removes effect of ordering r elements

Sep 30-10:47 AM

Ex. Five students (A, B, C, D, E) are running for student's council to fill positions of president, VP, and secretary.
(a) How many election results are possible?

P VP S
5 × 4 × 3 $= {}_5P_3$

$= 60$

$\therefore$ 60 ways to elect.

(b) How many ways could these 5 students form a 3-member committee?

C C C
5 × 4 × 3
3!

$= \frac{{}_5P_3}{3!}$

$= {}_5C_3$

$= 10$

$\therefore$ 10 ways to form committee.

Sep 30-11:26 AM

Ex. (a) How many ways to arrange a 5-cards on the table?
(b) How many arrangements of 5 cards can be dealt?
(c) How many ways are there to deal a 5-card hand?

(a) $5!$

(b) ${}_{52}P_5 = \frac{52!}{47!}$

$= 311\,875\,200$

$\frac{n!}{(n-r)!}$

(c) $\frac{{}_{52}P_5}{5!} = {}_{52}C_5$

$= 2\,598\,960$

Sep 30-11:03 AM

Ex. (b) How many ways are there to deal a 5-card hand?

Assume the cards dealt are: As, Kc, Qh, Jd, 10s.

The order these cards are received does not matter.

Identical hands

As, Kc, Qh, Jd, 10s
As, Qh, Kc, Jd, 10s
As, Kc, Qh, 10s, Jd
10s, As, Kc, Qh, Jd
Kc, Qh, Jd, 10s, As
10s, Jd, Qh, Kc, As
...

$${}_{52}C_5 = \frac{52!}{(52-5)!5!}$$

5! arrangements of the cards in the hand.

Sep 30-11:03 AM

Assigned Work:

Day1, p.279 # 3, 4, 6, 9, 11, 12

Day2, p.279 # 13, 14, 15, 17, 20, 22

9. (b) ${}_{11}C_3 = \frac{11!}{8!\,3!}$ $\quad {}_nC_r = \frac{n!}{(n-r)!\,r!}$

$= 165$

(c) $\frac{P}{11} \times \frac{T}{10} \times \frac{S}{9}$ $\quad {}_{11}P_3 = \frac{11!}{8!}$

$= 990$

ans C $\div 3!$ = ans B

(d) different by a factor of 6 or 3! because the order of 3 elements in the committee does not matter.

Sep 30-11:29 AM

11. ${}_{10}C_2 \times {}_8C_2 \times {}_7C_2$

$= 26460$

Feb 26-12:50 PM

13. (e) ${}_{15}C_5 + {}_{11}C_5$

mutually exclusive

14 bd

(b) ${}_{10}C_8 \times {}_{10}C_4$

playing 12 songs total

(d) 12 songs : ${}_{20}C_{12}$
OR
13 : ${}_{20}C_{13}$
OR
14 : ${}_{20}C_{14}$
OR
15 : ${}_{20}C_{15}$

Feb 27-2:02 PM

17. ${}_6C_3 \times {}_5C_2 \times {}_8C_4$
Y B W

20. # of hands: ${}_{52}C_{13} = \frac{52!}{39!\,13!}$

(a) # of arrangements: ${}_{52}P_{13} = \frac{52!}{39!}$

divide by 13! to remove the order of the 13-card hand.

(b) 5C 2S 3D 3H

${}_{13}C_5 \times {}_{13}C_2 \times {}_{13}C_3 \times {}_{13}C_3$

(c) 5H, 8 non-H

${}_{13}C_5 \times {}_{39}C_8$

Feb 27-2:08 PM