Problem Solving with Combinations

Feb 27/2020

Consider an experiment with N distinct elements:

1, 2, 3, ..., n

The set of elements would be: S = {1, 2, 3, ..., n}

We may also be interested in subsets of the set. A subset is any combination of elements from the set, from zero to the total number of elements, n.

For example, a subset of 3 elements could be

{1, 2, 3}, {1, 2, 4}, {2, 3, 4}, etc.

The total number of subsets would be $_nC_3$

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Ex. At the movies, they offer popcorn (P), chocolate (C), and soda (S). How many ways can you choose at least one treat?

① build all possible sets.

P, C, S $\quad {}_3C_1$

PC, PS, CS $\quad {}_3C_2$

PCS $\quad {}_3C_3$

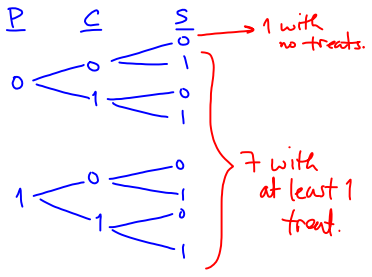
7

② popcorn: 0, 1 | choc: 0, 1 | soda: 0, 1

2 choices x 2 choices x 2 choices = 8 for popcorn

$8 - 1 = 7$

remove choosing no treats.



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The total number of possible subsets is a sum of all sets of all possible sizes.

$$ {}_nC_0 + {}_nC_1 + {}_nC_2 + ... + {}_nC_n $$

${}_nC_0 = 1$ is the number of null, or empty, sets.

The total number of subsets is also given by 2^n

The number of non-empty subsets is $2^n - 1$

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Why is the number of subsets 2^n ?

Consider each element in the set: S = {1, 2, 3, ..., n}

Each element can either be included in a subset, or excluded from a subset. Thus, each element has two options, or states, when forming subsets.

$$ S = \{1, 2, 3, ..., n\} $$

$$ 2 \times 2 \times 2 \times ... \times 2 = 2^n $$

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Ex. The break room needs snacks. Ravi can purchase some items without putting the order through accounting: up to 3 cases of cookies, 4 of juice, and 2 of tea. How many different purchases are possible?

C	J	T
0	0	0
1	1	1
2	2	2
3	3	
	4	

$4 \times 5 \times 3 = 60$

$60 - 1 = 59$

no snacks.

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When choosing between some, all, or none of p identical items, the number of choices is (p+1).

For multiple types, p items, q items, r items, etc., the total number of choices is

$$(p+1)(q+1)(r+1)...$$

Remember to think carefully about whether or not you want to include the null set, or no items. If you must choose <u>at least one</u> item

$$(p+1)(q+1)(r+1)... - 1$$

Assigned Work:

Day 1: p.286 # 1, 2, 3, 4, 5, 7, 8, 9, 10

Day 2: p.287-288 # 11, 12, 13, 16, 17, 18, 22

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Assigned Work:

Day 1: p.286 # 1, 2, 3, 4, 5, 7, 8, 9, 10

Day 2: p.287-288 # 11, 12, 13, 16, 17, 18, 22

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