Pascal's Triangle & the Binomial Theorem March 4/2020

The elements of each row can be related to combinations.

1	Row 0	${}_0C_0$
1 1	Row 1	${}_1C_0$ ${}_1C_1$
1 2 1	Row 2	${}_2C_0$ ${}_2C_1$ ${}_2C_2$
1 3 3 1	Row 3	${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$
1 4 6 4 1	Row 4	${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$

Recall: The sum of the elements for row 'n' is 2^n

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Pascal's Triangle & The Binomial Theorem

Consider:

$(x+y)^0 = 1$

$(x+y)^1 = 1x + 1y$

$(x+y)^2 = 1x^2 + 2xy + 1y^2$

$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$

$(x+y)^7 = 1x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3$
$+ 35x^3y^4 + 21x^2y^5 + 7xy^6 + 1y^7$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

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Ex. Write the 5th row of Pascal's triangle:
(a) using combination notation.
(b) as numbers.

$\frac{5!}{3!2!}$

(a) ${}_5C_0$ ${}_5C_1$ ${}_5C_2$ ${}_5C_3$ ${}_5C_4$ ${}_5C_5$

$\frac{5!}{2!3!}$

(b) 1 5 10 10 5 1

Ex. Write the first four terms from row 14 using:
(a) combination notation.
(b) numbers.

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

(a) ${}_{14}C_0$ ${}_{14}C_1$ ${}_{14}C_2$ ${}_{14}C_3$

(b) 1 14 91 364

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The coefficients from the *binomial expansions* can be organized using Pascal's triangle.

$$(a+b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{n-1} b^1 + \ldots + {}_nC_n a^0 b^n$$

<u>Summation notation</u> can be used to express this in a more concise manner.

$$(a+b)^n = \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

start counting from r = 0,
and stop at r = n.

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Ex. Write using summation notation, then expand and simplify.

$(2x-3y)^5$

$(a+b)^n = \sum_{r=0}^{n} {}_nC_r\, a^{n-r} b^r$

$= \sum_{r=0}^{5} {}_5C_r (2x)^{5-r}(-3y)^r$

$= {}_5C_0(2x)^5(-3y)^0 + {}_5C_1(2x)^4(-3y)^1 + {}_5C_2(2x)^3(-3y)^2$

$+ {}_5C_3(2x)^2(-3y)^3 + {}_5C_4(2x)^1(-3y)^4 + {}_5C_5(2x)^0(-3y)^5$

$= (1)(32x^5)(1) + (5)(16x^4)(-3y) + 10(8x^3)(9y^2)$

$+ 10(4x^2)(-27y^3) + 5(2x)(81y^4) + 1(1)(-243y^5)$

$= 32x^5 - 240x^4y + 720x^3y^2 - 1080x^2y^3 + 810xy^4 - 243y^5$

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Assigned Work:

p.293 # 2ac, 3, 4, 5abc, 9ace, 11ac

Recommended Review:

p.296 # 1, 2, 3, 4cgk, 5, 6, 8-13, 14, 15a, 16d,
17, 19, 20,21, 23

p.298 # 5 - 9

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5 bc

5(b) ${}_{12}C_0 - {}_{12}C_1 + {}_{12}C_2 \; \dots - {}_{12}C_{11} + {}_{12}C_{12}$

_ _ _ _ _ _ _ _ _ _ _ _ _ row 12

$2\,{}_{12}C_0 - 2\,{}_{12}C_1 + 2\,{}_{12}C_2 - 2\,{}_{12}C_3 + 2\,{}_{12}C_4 - 2\,{}_{12}C_5$
$+ 1\,{}_{12}C_6$

(c) $\sum_{r=0}^{15} {}_{15}C_r$

$= {}_{15}C_0 + {}_{15}C_1 + \dots + {}_{15}C_{14} + {}_{15}C_{15}$

$= 2^{15}$

0 1
1 1 1
2 +1 −2 +1
3 +1 −3 +3 −1
4 +1 −4 +6 −4 +1
5 +1 −5 +10 −10 +5 −1
6 +1 −6 +15 −20 +15 −6 +1

∴ even rows, alternating +/− adds to zero

∴ also true for odd rows.

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11(a)

$\left(x^2 - \frac{1}{x}\right)^5$

$= {}_5C_0 (x^2)^5 \left(\frac{-1}{x}\right)^0 + {}_5C_1 (x^2)^4 \left(\frac{-1}{x}\right)^1 + {}_5C_2 (x^2)^3 \left(\frac{-1}{x}\right)^2$
$+ {}_5C_3 (x^2)^2 \left(\frac{-1}{x}\right)^3 + {}_5C_4 (x^2)^1 \left(\frac{-1}{x}\right)^4 + {}_5C_5 (x^2)^0 \left(\frac{-1}{x}\right)^5$

$= x^{10} + 5x^8\left(-\frac{1}{x}\right) + 10x^6\left(\frac{1}{x^2}\right)$
$+ 10x^4\left(\frac{-1}{x^3}\right) + 5x^2\left(\frac{1}{x^4}\right) + 1\left(\frac{-1}{x^5}\right)$

$= x^{10} - 5x^7 + 10x^4 - 10x + \frac{5}{x^2} - \frac{1}{x^5}$

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5 (c)

$$\sum_{r=0}^{15} {}_{15}C_r = {}_{15}C_0 + {}_{15}C_1 + {}_{15}C_2 + \cdots + {}_{15}C_{15}$$

$$= 2^{15}$$

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