

## Pascal's Triangle & the Binomial Theorem March 4/2020

The elements of each row can be related to combinations.

1	Row 0	$_0C_0$
1 1	Row 1	$_1C_0 \quad _1C_1$
1 2 1	Row 2	$_2C_0 \quad _2C_1 \quad _2C_2$
1 3 3 1	Row 3	$_3C_0 \quad _3C_1 \quad _3C_2 \quad _3C_3$
1 4 6 4 1	Row 4	$_4C_0 \quad _4C_1 \quad _4C_2 \quad _4C_3 \quad _4C_4$

Recall: The sum of the elements for row 'n' is  $2^n$

Oct 2-6:47 PM

## Pascal's Triangle & The Binomial Theorem

Consider:

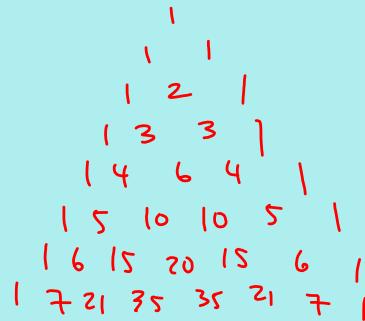
$$(x+y)^0 = 1$$

$$(x+y)^1 = 1x + 1y$$

$$(x+y)^2 = 1x^2 + 2xy + 1y^2$$

$$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$$

$$(x+y)^7 = 1x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + 1y^7$$



May 30-10:48 PM

Ex. Write the 5th row of Pascal's triangle:

(a) using combination notation.

(b) as numbers.

$$(a) \quad {}_5C_0 \quad {}_5C_1 \quad {}_5C_2 \quad {}_5C_3 \quad {}_5C_4 \quad {}_5C_5$$

$$(b) \quad 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$$

$$\frac{5!}{3!2!}$$

$$\frac{5!}{2!3!}$$

Ex. Write the first four terms from row 14 using:

(a) combination notation.

(b) numbers.

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

$$(a) \quad {}_{14}C_0 \quad {}_{14}C_1 \quad {}_{14}C_2 \quad {}_{14}C_3$$

$$(b) \quad 1 \quad 14 \quad 91 \quad 364$$

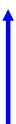
Mar 4-1:28 PM

The coefficients from the *binomial expansions* can be organized using Pascal's triangle.

$$(a+b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{n-1} b^1 + \dots + {}_nC_n a^0 b^n$$

Summation notation can be used to express this in a more concise manner.

$$(a+b)^n = \sum_{r=0}^n {}_nC_r a^{n-r} b^r$$



start counting from  $r = 0$ ,  
and stop at  $r = n$ .

May 30-10:48 PM

Ex. Write using summation notation, then expand and simplify.

$$\begin{aligned}
 & (2x - 3y)^5 \\
 &= \sum_{r=0}^5 {}_5 C_r (2x)^{5-r} (-3y)^r \quad (a+b)^n = \sum_{r=0}^n {}_n C_r a^{n-r} b^r \\
 &= {}_5 C_0 (2x)^5 (-3y)^0 + {}_5 C_1 (2x)^4 (-3y)^1 + {}_5 C_2 (2x)^3 (-3y)^2 \\
 &\quad + {}_5 C_3 (2x)^2 (-3y)^3 + {}_5 C_4 (2x)^1 (-3y)^4 + {}_5 C_5 (2x)^0 (-3y)^5 \\
 &= (1)(32x^5)(1) + (5)(16x^4)(-3y) + 10(8x^3)(9y^2) \\
 &\quad + 10(4x^2)(-27y^3) + 5(2x)(81y^4) + 1(1)(-243y^5) \\
 &= 32x^5 - 240x^4y + 720x^3y^2 - 1080x^2y^3 + 810xy^4 - 243y^5
 \end{aligned}$$

Mar 4-8:35 AM

Assigned Work:

p.293 # 2ac, 3, 4, 5abc ~~9ace~~ 11ac

Recommended Review:

p.296 # 1, 2, 3, 4cgk, 5, 6, 8-13, 14, 15a, 16d,  
17, 19, 20, 21, 23

p.298 # 5 - 9

Mar 4-8:35 AM

5 bc

$$\begin{aligned}
 5(s) \quad & {}_{12}C_0 - {}_{12}C_1 + {}_{12}C_2 - \dots - {}_{12}C_{11} + {}_{12}C_{12} \\
 & \quad \cdots \cdots \cdots \text{row 12} \\
 & {}_2C_0 - 2{}_2C_1 + 2{}_2C_2 - 2{}_2C_3 + 2{}_2C_4 - 2{}_2C_5 \\
 & \quad \quad \quad \quad \quad \quad + 1{}_2C_6 \\
 (c) \quad & \sum_{r=0}^{15} {}_{15}C_r \\
 = & {}_{15}C_0 + {}_{15}C_1 + \dots + {}_{15}C_{14} + {}_{15}C_{15} \\
 = & 2^{15}
 \end{aligned}$$


---

0		1					
1		1	1				
2	+1	-2	+1				
3	+1	3	+3	-1			
4	+1	4	+6	-4	+1		
5	+1	-5	+10	-10	+5	-1	
6	+1	-6	+15	-20	+15	-6	+1

∵ even rows,  
 alternating +/-  
 adds to zero  
 ∴ also true for  
 odd rows.

Mar 5-2:05 PM

II(a)

$$\begin{aligned}
 & \left(x^2 - \frac{1}{x}\right)^5 \\
 = & {}_5C_0 (x^2)^5 \left(\frac{-1}{x}\right)^0 + {}_5C_1 (x^2)^4 \left(\frac{-1}{x}\right)^1 + {}_5C_2 (x^2)^3 \left(\frac{-1}{x}\right)^2 \\
 & + {}_5C_3 (x^2)^2 \left(\frac{-1}{x}\right)^3 + {}_5C_4 (x^2)^1 \left(\frac{-1}{x}\right)^4 + {}_5C_5 (x^2)^0 \left(\frac{-1}{x}\right)^5 \\
 = & x^{10} + 5x^8 \left(\frac{-1}{x}\right) + 10x^6 \left(\frac{1}{x^2}\right) \\
 & + 10x^4 \left(\frac{-1}{x^3}\right) + 5x^2 \left(\frac{1}{x^4}\right) + 1 \left(\frac{-1}{x^5}\right) \\
 = & x^{10} - 5x^7 + 10x^4 - 10x + \frac{5}{x^2} - \frac{1}{x^5}
 \end{aligned}$$

Mar 5-2:14 PM

5 (c)

$$\sum_{r=0}^{15} {}_{15}C_r = {}_{15}C_0 + {}_{15}C_1 + {}_{15}C_2 + \dots + {}_{15}C_{15}$$
$$= 2^{15}$$

Mar 5-2:20 PM