## Basic Probability

March 10/2020

Probability is a measure of the likelihood that an event will occur. It is always a value between 0 and 1 (or 0% to 100%).

Subjective probability is a statement made based onintuition, often with little or no mathematical data.

Experimental probability is a calculation based on the data obtained from a number of trials.

Theoretical probability is a purely mathematical analysis based upon all possible outcomes for the system.

Note: In general, when the term "probability" is used with no context, we will be referring to "theoretical probability." If you are unsure, ask!

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Experimental, or Empirical, Probability:

In a probability experiment, we count the number of outcomes we are looking for (A) compared to the total number of trials (T).

$$P(A) = \frac{n(A)}{n(T)}$$

number of occurences of A

total number of trials

As the number of trials increases, the quality of the experimental probability improves.

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Any experiment worth performing will have multiple possible outcomes. Adding all of these will always be 1, which can also mean 100%.

For an experiment with N outcomes:

$$P_1 + P_2 + P_3 + \ldots + P_N = 1$$

Ex. A coin is flipped 5 times, landing on 'heads' one time. Determine:
(a) P(H)
(b) P(T)
(c) P(H) + P(T)

(a) $P(H) = \frac{1}{5}$
$= 0.2$

(b) $P(T) = \frac{4}{5}$
$= 0.8$

(c) $P(H) + P(T) = \frac{1}{5} + \frac{4}{5}$
$= \frac{5}{5}$
$= 1$

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Theoretical Probability

Theoretical probability is determined by analyzing all possible outcomes of an event. The set of all possible outcomes is called the sample space for the event.

For equally likely outcomes:

$$P(A) = \frac{n(A)}{n(S)}$$

$$\text{probability of A} = \frac{\text{number of outcomes resulting in A}}{\text{total outcomes in sample space}}$$

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A sample space is often represented using set notation.

For example, when rolling a six-sided die:

odd odd odd

$$S = \{1, 2, 3, 4, 5, 6\}$$

$1^2=1$ $2^2=4$

Ex. For a single die, what is the probability of:
(a) rolling an odd number?
(b) rolling a perfect square?

(a) $P(\text{odd}) = \frac{n(\text{odd})}{n(S)} = \frac{3}{6} = 0.5 = \frac{1}{2}$

(b) $P(\text{sq.}) = \frac{n(\text{sq.})}{n(S)} = \frac{2}{6} = \frac{1}{3}$

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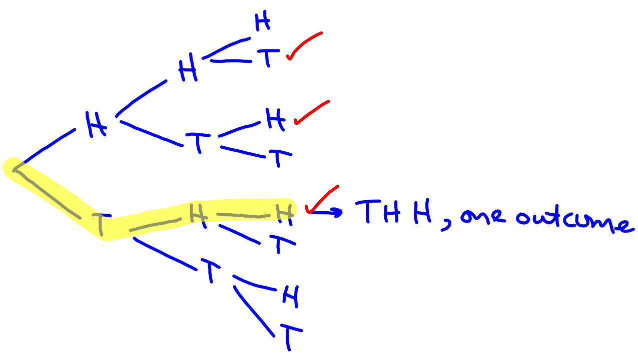
Some experiments are complex enough that more work is required to identify all possible outcomes.
A tree diagram, or decision tree, can be used to organize the information.

Ex. Use a tree diagram to determine:
(a) all possible outcomes for flipping three coins.
(b) the probability of getting exactly two heads.




(a) 8 possible outcomes $(2^N$, N coin flips$)$

(b) $P(2H) = \frac{3}{8}$

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For event A, we define A' (or ~A) as the complement of A. The complement includes all events which are not A.

$$P(A) + P(A') = 1$$

probability of event A

probability of any event not A

probability of an event

Ex. The game of battleship is played on a 10x10 board. Determine the probability of:
(a) hitting a ship on the first turn.
(b) missing a ship on the first turn.

$n(S) = 100$ possible targets
$n(\text{hits}) = 16$

(a) $P(\text{hit on turn 1}) = \frac{16}{100}$
$= \frac{4}{25}$

(b) $P(A) + P(A') = 1$
$P(\text{hit}) + P(\text{miss}) = 1$
$P(\text{miss}) = 1 - \frac{4}{25}$
$= \frac{25}{25} - \frac{4}{25}$
$= \frac{21}{25}$

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In battleship, you place 5 ships on a board and your opponent tries to 'hit' your ships by guessing their location.

Defensive Grid

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| A | | | | | | | | | | |
| B | | | | | | | | | | |
| C | | | | | | | | | | |
| D | | | | | | | | | | |
| E | | | | | | | | | | |
| F | | | | | | | | | | |
| G | | | | | | | | | | |
| H | | | | | | | | | | |
| I | | | | | | | | | | |
| J | | | | | | | | | | |

Put the following ships on your defensive grid by placing the appropriate letters -- horizontally, vertically or diagonally.

1 - Aircraft Carrier
A A A A A

1 - Battleship
B B B B

1 - Cruiser
C C C

2 - Destroyers
D D   D D

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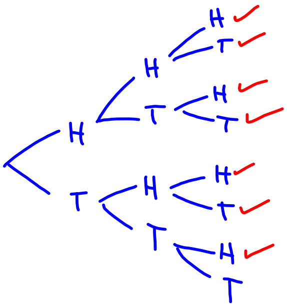
Assigned Work:

p.312 # 1, 2, 4, 6, 7, 10, 11, 12

e c b

1. (c)




(c) $P(\geq 1H) = \frac{7}{8}$

OR

$P(\geq 1H) = 1 - P(0H)$
$= 1 - P(3T)$
$= 1 - \frac{1}{8}$
$= \frac{7}{8}$

(e)

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

P(not PS)
$= 1 - P(PS)$
$= 1 - \frac{7}{36}$
$= \frac{36}{36} - \frac{7}{36}$
$= \frac{29}{36}$

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7. $A = 5, 7, 9$ $n(A) = 14$
$B = A'$ $n(B) = 22$

$\therefore$ B has advantage, more outcomes result in B winning

OR $P(A\text{ win}) = \frac{14}{36} = \frac{7}{18}$ $P(B\text{ win}) = \frac{22}{36} = \frac{11}{18}$

(b) $A = 5, 7$ or doubles
$n(A) = 16$
$n(B) = 20$

$\therefore$ B still has the advantage.

(c)

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12..

| | |
|---|---|
| A | 15 |
| B | 25 |
| C | 30 |
| D | 15 |
| E | 15 |

(a) always pick 'C'

(b)

| | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| A | 15 | | | | | |
| B | 25 | | 10 | | | |
| C | 30 | ✓10 | | | | |
| D | 15 | | | | | |
| E | 15 | | | | | |

C B C B C

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