Probabilities Using Counting Techniques March 12/2020

We can use counting techniques, such as permutations and combinations, to determine the values required to calculate many probabilities.

$$P(A) = \frac{n(A)}{n(all)}$$

Ex. Two brothers enter a race with five friends. What is the probability that:
(a) the older is in lane 1 and younger in lane 2?
(b) they are next to each other?

(a) $n(L1, L2) = 1$ $\quad n(all) = {}_7P_7 = 7!$

$P(L1, L2) = \frac{1}{7!}$

$= \frac{1}{5040}$

(b) $n(\text{brothers adjacent}) = {}_6P_6 \times {}_2P_2$

$= 1440$

_ _ _ _ _ _

B_1B_2

$P(\text{adjacent}) = \frac{144\cancel{0}}{504\cancel{0}}$

$= \frac{144}{504}$

$= \frac{72}{252}$

$= \frac{36}{126}$

$= \frac{18}{63}$

$= \frac{2}{7}$

Ex. A focus group of three is selected from five doctors and seven technicians. What is the probability of:
(a) doctors only?
(b) at most one doctor?

$n(all) = {}_{12}C_3$

(a) $n(all\ D) = \underbrace{{}_5C_3}_{\text{choose 3 of 5 docs}} \cdot \underbrace{{}_7C_0}_{\text{choose 0 of 7 techs}}$

$P(3\ D) = \frac{{}_5C_3}{{}_{12}C_3}$

$= \frac{1}{22}$

(b) $n(\leq 1D) = \underbrace{{}_5C_1 \cdot {}_7C_2}_{1D} + \underbrace{{}_5C_0 \cdot {}_7C_3}_{0D}$

$P(\leq 1D) = \frac{140}{220}$

$= \frac{7}{11}$

Ex. What is the probability in a class of 24 that two students will share the same birthday?

$S_1 \quad S_2 \quad S_3$

$365 \quad 364 \quad 363$

$n(\text{non-matching days}) = 365 \times 364 \times 363 \times \ldots$

$= {}_{365}P_{24}$

$n(\text{all}) = (365)^{24}$

$$P(\text{none same}) = \frac{n(\text{not same})}{n(\text{all})} = \frac{{}_{365}P_{24}}{(365)^{24}} \doteq 0.462$$

$$P(\geq 1 \text{ same}) = 1 - P(0 \text{ same}) \doteq 1 - 0.462 \doteq 0.538$$

∴ in a class of 24, there is a 53.8% chance of the same birthday

Assigned Work:

p.324 # 1, 2, 3, 5, 6, 7, 8, 9, 11, 13, 15, 16

13(a) 3702

(b) $\frac{1}{3960}$ ✓

PROBABILITY 11 letters, 9 unique

4-letters?

case 1: all unique ${}_9P_4$ → "arrangements"

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2: one pair, BB XY or II $\left(\frac{4!}{2!}\right)({}_8C_1)({}_7C_1)(2)$ 2 unique pairs

\+

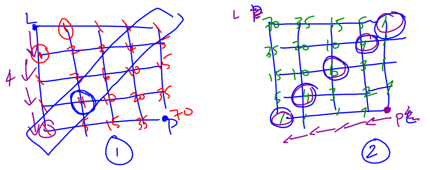
3: 2 pairs BBII $\frac{4!}{2!\,2!}$

(b) $P(\text{"BARB"}) = \frac{n(\text{"BARB"})}{n(\text{all})}$ 2

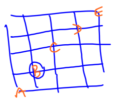
not answer to (a)

$n(\text{all}) = {}_{11}P_4$ $n(\text{BARB}) = (2)(1)(1)(1) = 2$

$$P(\text{BARB}) = \frac{2}{{}_{11}P_4} = \frac{2}{7920} = \frac{1}{3960}$$



regardless of path, only meet after 4 "steps"



$n_{1A} = 1$ $n_{2A} = 1$

$n_{1B} = 4$ $n_{2B} = 4$

$n_{1C} = 6$ $n_{2C} = 6$

$n_{1D} = 4$ $n_{2D} = 4$

$n_{1E} = 1$ $n_{2E} = 1$

$n(\text{meet } A) = (1)(1) = 1$

$n(\text{meet } B) = (4)(4) = 16$

$n(\text{meet } C) = (6)(6) = 36$

$n(\text{meet } D) = (4)(4) = 16$

$n(\text{meet } E) = (1)(1) = 1$

$n(\text{meet}) = 70$

$n(1 \text{ middle}) = 1+4+6+4+1$
$= 16$

$n(2 \text{ middle}) = 16$

$n(\text{all to middle}) = (16)(16)$
$= 256$

$p(\text{meet}) = \frac{70}{256}$
$= \frac{35}{128}$

16. 8N, 2D $\quad P(\sim \text{all } N) \geq 90\%$

OR

$\underbrace{P(\text{all } N) < 10\%}$ $= 0.10$

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try size 5 $\quad n(\text{all } N) = {}_8C_5$
$= 56$

$P(\text{all}N) = \frac{56}{252}$ $\quad n(\text{all}) = {}_{10}C_5$

$\doteq 0.22$ $\quad = 252$

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try 6 $\quad n(\text{all}N) = {}_8C_6$
$= 28$

$P(\text{all}N) = \frac{28}{210}$

$\doteq 0.13$ $\quad n(\text{all}) = {}_{10}C_6$
$= 210$

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try 7 ✓ $\quad P(\text{all } N) = \frac{{}_8C_7}{{}_{10}C_7}$
$= 0.067$ ✓

$\therefore$ it needs to be at least 7 people.