Rational Functions Graphs

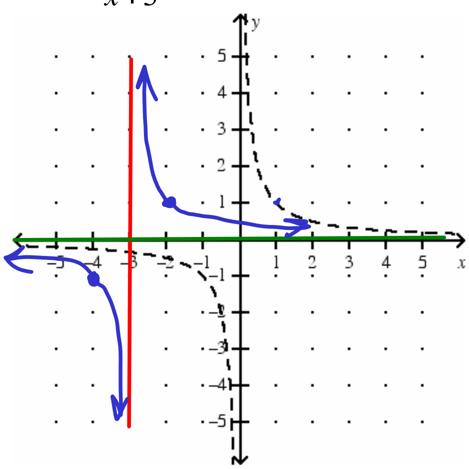
In the functions unit we studied the Reciprocal Function, which is in the family of Rational Functions.

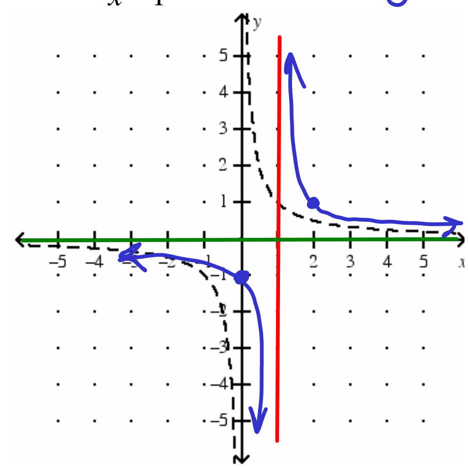
We looked at $y=\frac{1}{x}$

Use your knowledge of functions (and transformations) to sketch the graph of each of the following.

a) $y=\frac{1}{x+3}$ horizontal shift left 3

b) $y=\frac{1}{x-1}$ h. shift right by 1





Remember: The zeros of the denominator result in vertical asymptotes.

* What happens when a factor of the denominator is also a factor of the numerator?

If a factor in the denominator divides out with the same factor in the numerator, the restriction takes the shape of a hole in the graph.

If a factor in the denominator does not divide out, the restriction is a vertical asymptote.

Ex: Simplify the equation of each of the functions, decide whether you have a hole and/or a vertical asymptote, and sketch the graph of the function.

a) $y=\frac{3x-1}{3x^2+5x-2}$

$=\frac{3x-1}{(x+2)(3x-1)}$

$=\frac{1}{x+2}$, VA $x \neq -2$

$x \neq \frac{1}{3}$

M -6
A 5
N $\frac{6}{3}, \frac{-1}{3x}$
↓
$\frac{2}{1x}$

$x+2=0$

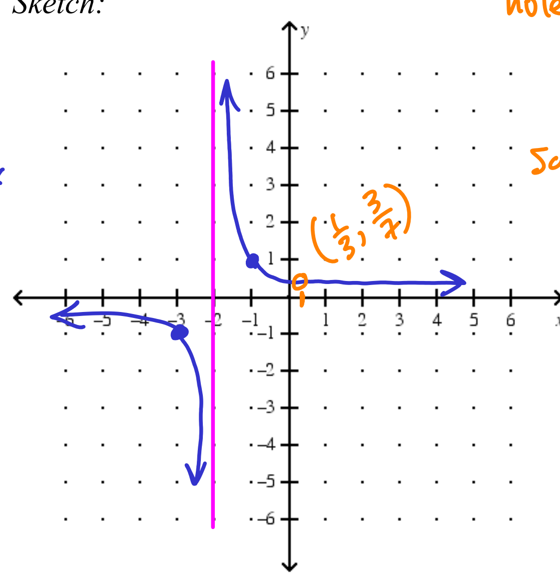
$x=-2$

$3x-1=0$

$3x=1$

$x=\frac{1}{3}$

Sketch:



hole when $x=\frac{1}{3}$

sub in

$y=\frac{1}{x+2}$

$=\frac{1}{\frac{1}{3}+2}$

$=\frac{1}{\frac{1}{3}+\frac{6}{3}}$

$=\frac{1}{\frac{7}{3}}$

$=\frac{3}{7}$

hole at $(\frac{1}{3}, \frac{3}{7})$

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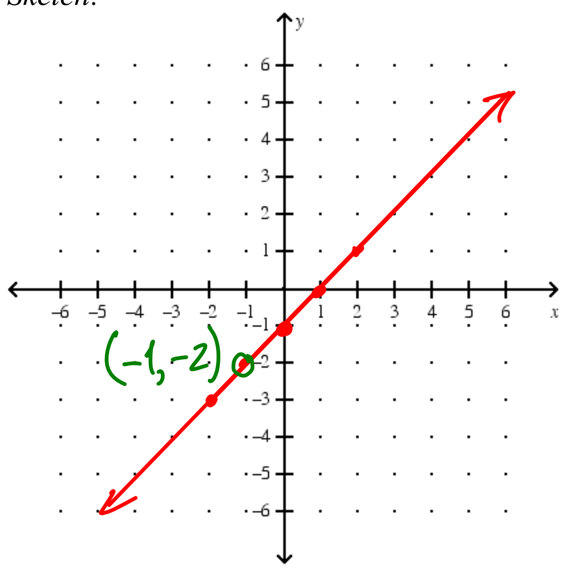
b) $y=\dfrac{x^2-1}{x+1}$

$= \dfrac{(x-1)\cancel{(x+1)}}{\cancel{x+1}}$

$= \underbrace{x-1}$, $x \neq -1$

straight line

Sketch:



hole at

$x=-1$

$y = x-1$

$= (-1)-1$

$= -2$

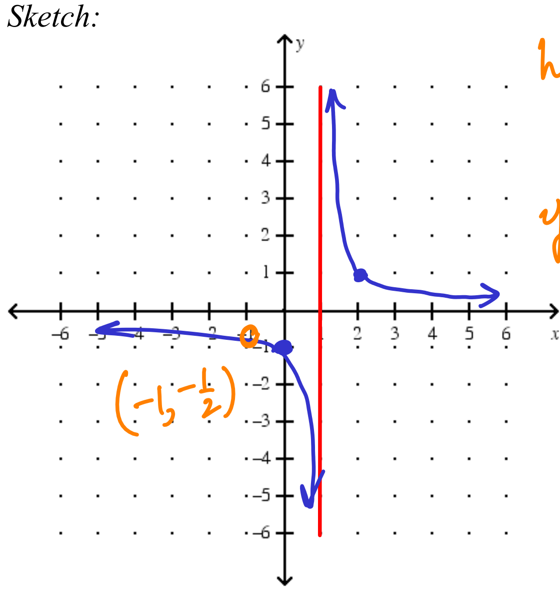
c) $y=\dfrac{x+1}{x^2-1}$

$= \dfrac{\cancel{x+1}}{(x-1)\cancel{(x+1)}}$

$= \dfrac{1}{x-1}$, $x \neq 1$, $x \neq -1$

VA hole

Sketch:



hole at

$x=-1$

$y = \dfrac{1}{x-1}$

$= \dfrac{1}{-1-1}$

$= \dfrac{1}{-2}$

$= -\dfrac{1}{2}$

d) $y=\dfrac{2x^2-7x+6}{x-2}$

$= \dfrac{(2x-3)\cancel{(x-2)}}{\cancel{x-2}}$

$= 2x-3$, $x \neq 2$

hole

M 12

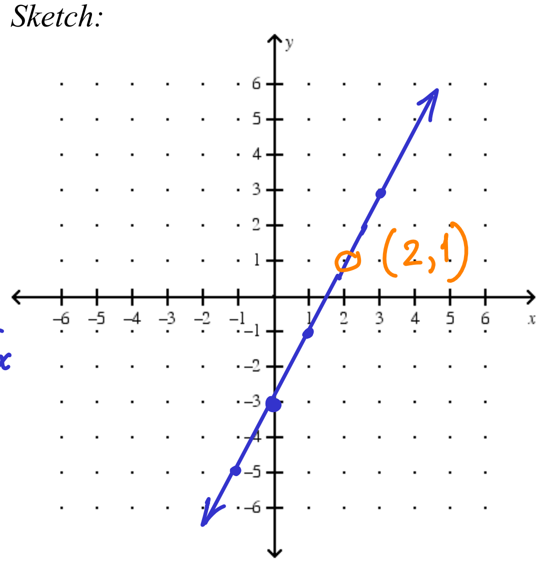
A −7

N $\dfrac{-3}{2x}$ $\dfrac{-4}{2}$

↓

$\dfrac{-2}{1x}$

Sketch:



$y = 2(2)-3$

$= 4-3$

$= 1$

(2,1)

Rational Functions Graphs

e) $y=\dfrac{x^2-9}{4x+12}$

$= \dfrac{(x-3)(x+3)}{4(x+3)}$

$= \dfrac{x-3}{4}, \; x \neq -3$

$= \frac{1}{4}(x-3), \; x \neq -3$

$= \frac{1}{4}x - \frac{3}{4}, \; x \neq -3$

$M = \frac{1}{4}$

$y\text{-int} = \frac{-3}{4}$

$x\text{-int} = 3$

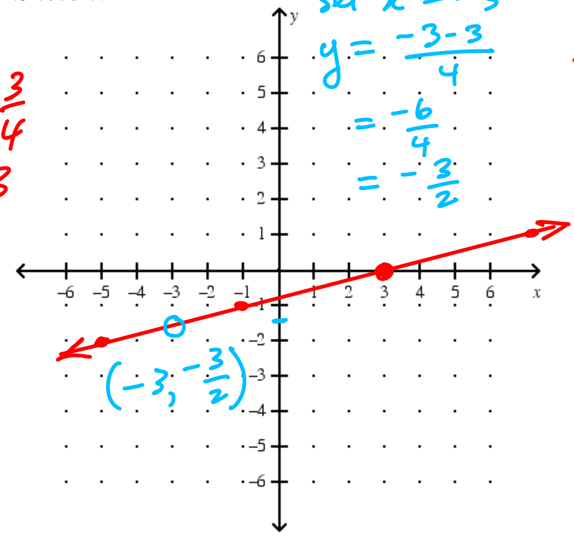
Sketch:

set $x = -3$

$y = \dfrac{-3-3}{4}$

$= \dfrac{-6}{4}$

$= -\dfrac{3}{2}$



x-int?

set $y=0$

$y = \dfrac{x-3}{4}$

$0 = \dfrac{x-3}{4}$

$0 = x-3$

$3 = x$

f) $y=\dfrac{4x-10}{4x^2-25}$

$= \dfrac{2(2x-5)}{(2x-5)(2x+5)}$

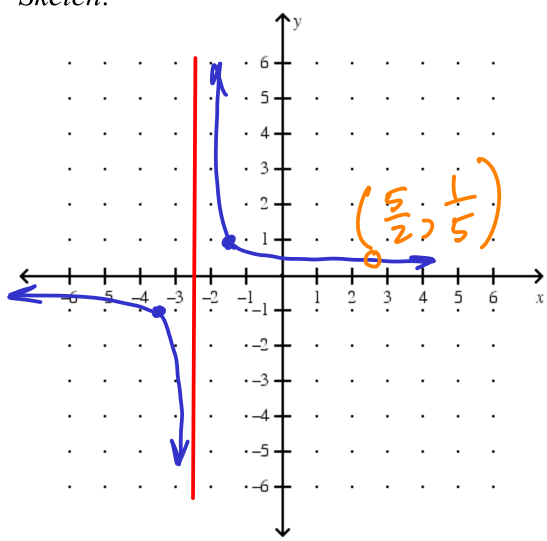
$= \dfrac{2}{2x+5}, \; x \neq -\frac{5}{2}$ VA

$x \neq \frac{5}{2}$ hole

$= \dfrac{2}{2(x+\frac{5}{2})} \qquad x \neq \pm\frac{5}{2}$

$= \dfrac{1}{x+\frac{5}{2}} \qquad x \neq \pm\frac{5}{2}$

Sketch:



hole at $x = \frac{5}{2} = 2.5$

$y = \dfrac{2}{2x+5}$

$= \dfrac{2}{2(\frac{5}{2})+5}$

$= \dfrac{2}{10} = \dfrac{1}{5}$