Quadratic Relations in Factored Form

Key Concepts:

- factored form of quadratic relation
- direction of opening from 'a'
- solving for zeroes

- using symmetry to find:
  - x-coordinate of vertex
  - axis of symmetry

- using substitution to find:
  - y-coordinate of vertex
  - y-intercept

Apr 10-6:32 PM

Is $y = 2(x+1)(x-5)$ a quadratic relation?

Examine 1st and 2nd differences:

x	y
-2	14
-1	0
0	-10
1	-16
2	-18

$0-14=-14$

$-10-0=-10$

$-16-(-10)=-6$

$-18-(-16)=-2$

$-10-(-14)=4$

$-6-(-10)=4$

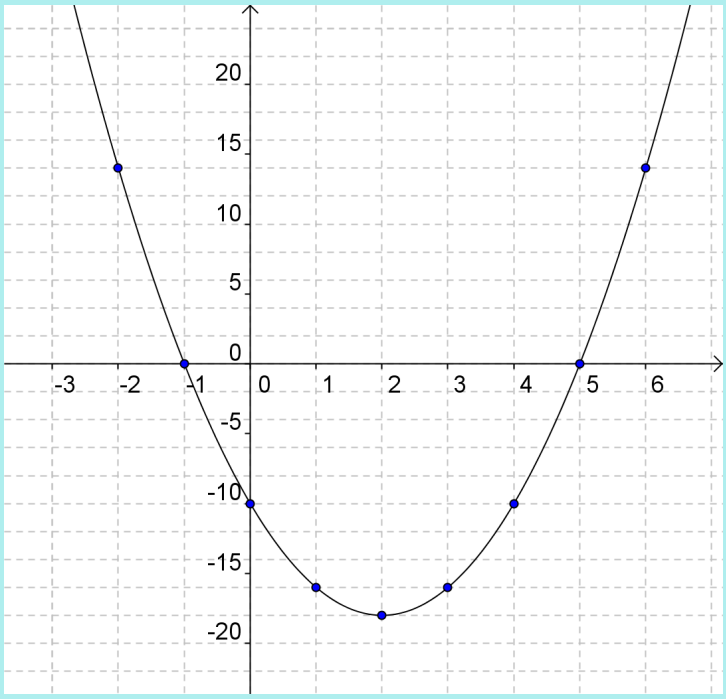
$-2-(-6)=4$

Oct 19-8:29 PM

Is $y = 2(x+1)(x-5)$ a quadratic relation?

Graph the relation:

x	y
-2	14
-1	0
0	-10
1	-16
2	-18
3	-16
4	-10
5	0
6	14



Oct 19-8:29 PM

Quadratic Relations in Factored Form

Oct 20/2011

The equation of a quadratic relation may be written in several forms:

1. standard form: $y = ax^2 + bx + c$
2. factored form: $y = a(x - s)(x - t)$
3. vertex form: $y = a(x - h)^2 + k$

The factored form, $y = a(x - s)(x - t)$, is most useful for finding the zeroes, which are $x = s$ and $x = t$.

Mar 20 - 4:17 PM

Consider the following...

Give two numbers that have a product of zero:

$6 \times 0 = 0$ $-2 \times 0 = 0$ $0 \times 0 = 0$

What do you notice? $(\text{any value}) \times 0 = 0$

Solve:

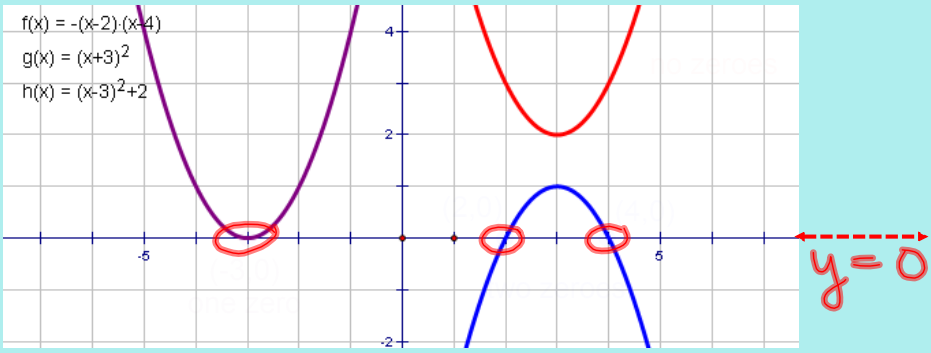
(a) $3x = 0$ $\frac{3x}{3} = \frac{0}{3}$ $x = 0$

(b) $57y = 0$ $y = 0$

(c) $3xy = 0$ $x = 0$ OR $y = 0$

Mar 31-8:45 AM

Depending upon the location of the vertex, and whether the parabola opens up or down, it may have 0, 1, or 2 distinct (unique) zeroes.



Zeroes occur where the y-coordinate of the parabola is equal to zero.

Apr 17-11:18 PM

To find the zeroes algebraically, we **set y = 0** and solve for the x-values that make the equation true.

Ex.1 Determine the zero(es) of each

(a) $y = x(x - 10)$

Set $y = 0$

$0 = x(x-10)$

$x = 0$ or $x - 10 = 0$

$x = 10$

Recall:

Zero multiplied by anything is zero.

If (a)(b) = 0 then
a = 0 or b = 0 (or both are zero).

(b) $y = -2(x - 5)(3x - 1)$

Set $y = 0$

$0 = -2(x-5)(3x-1)$

$x - 5 = 0$ or $3x - 1 = 0$

$x = 5$ $3x = 1$

$x = \frac{1}{3}$

(c) $y = 2(x - 2)^2$

Set $y = 0$

$0 = 2(x-2)^2$

$0 = 2(x-2)(x-2)$

$x - 2 = 0$ or $x - 2 = 0$

$x = 2$ $x = 2$

Apr 17-11:30 PM

The zeroes and symmetry can be used to find the vertex (h, k).

For the x-coordinate (h), find the midpoint of the zeroes:

$$MP_x = \frac{x_1 + x_2}{2} = \frac{s + t}{2}$$

For the y-coordinate (k), substitute the midpoint into the equation and solve for y:

$y = a(x - s)(x - t)$

$y = a(MP - s)(MP - t)$

Apr 17-11:45 PM

Ex.2 Determine the vertex:

(a) $y = -2(x - 2)(x - 8)$ predict zeroes: 2, 8

① Set $y = 0$

$0 = -2(x-2)(x-8)$

$x-2=0$ or $x-8=0$

$x=2$ $x=8$

② $x_m = \frac{2+8}{2}$

$x_m = 5$

③ Sub $x_m = 5$ into equation

$y = -2(5-2)(5-8)$

$= -2(3)(-3)$

$= -2(-9)$

$= 18$

∴ vertex is $(5, 18)$

Apr 18-12:03 AM

Ex.3 A parabola has zeroes at -3 (s) and 2 (t), and a y-intercept of 18. Determine the equation.

$y = a(x-s)(x-t)$

$y = a(x-(-3))(x-(2))$

$y = a(x+3)(x-2)$

y-int 18 $\rightarrow (0, 18)$

Sub (0,18) to find a

$18 = a(0+3)(0-2)$

$18 = a(3)(-2)$

$\frac{18}{-6} = \frac{a(-6)}{-6}$

$-3 = a$

∴ $y = -3(x+3)(x-2)$

Oct 19-9:26 PM

Assigned Work:

p. 155-157 # 2, 3, 4ace, 5, 6ace, 7, 10