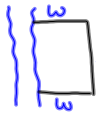Applications of Quadratics in Factored Form Nov 4/2011

Note: All examples from lesson are included in handout.
You only need to write out solutions.

1. A rectangular lot is bounded on one side by a river and on the other three sides by 80m of fencing. Determine the dimensions of the largest lot possible.

maximum area

$A = \ell \times w$ ①

$w + \ell + w = 80$
$\ell + 2w = 80$ ②

for substitution, need to isolate a variable

②: $\ell = 80 - 2w$ ②

Sub ② into ①
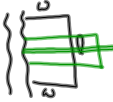$A = \ell \cdot w$
$A = (80 - 2w) \cdot w$ ← quadratic equation in factored form

Want max Area → vertex
use zeroes to find vertex.
Set $A = 0$
$0 = (80 - 2w) \cdot w$

$80 - 2w = 0$ or $\boxed{w = 0}$
$80 = 2w$
$\boxed{40 = w}$

to find w for max area
$w = \dfrac{40 + 0}{2}$ (axis of symmetry)
$w = 20$
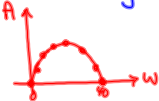
sub $w = 20$ into ②: $\ell = 80 - 2w$
$\ell = 80 - 2(20)$
$\ell = 80 - 40$
$\ell = 40$

∴ dimensions for max area are 20m by 40m

Apr 19-7:41 PM

2. Supermarket cashiers try to memorize current sale prices while they work. A study showed that, on average, the percent, P, of prices memorized after t hours is given approximately by the formula

$$P = -40t^2 + 120t$$

What is the greatest percent of prices memorized, and how long does it take to memorize them?

$P = -40t^2 + 120t$

$P = -40t(t - 3)$

for zeroes, set $P = 0$

$0 = -40t(t - 3)$

$-40t = 0$ or $t - 3 = 0$
$t = 0$ $t = 3$

for t vertex, $t_v = \dfrac{0 + 3}{2}$
$t_v = 1.5$

∴ study for 1.5 hours to maximize amount memorized

Sub $t = 1.5$ in $P = -40t^2 + 120t$
$= -40(1.5)^2 + 120(1.5)$
$= -40(2.25) + 180$
$= -90 + 180$
$= 90$

∴ the max memorized is 90% in 1.5 hours

Apr 20-5:48 PM

3. The cost of a ticket to a hockey arena is $3, and the arena holds 800 people. At this price, every ticket is sold. A survey indicates that for every dollar increase in price, attendance will fall by 100 people.
    (a) What ticket price results in the greatest revenue?
    (b) What is the greatest revenue?

$$\text{Revenue} = (\text{price})(\#\text{ sold})$$

$$2400 = (3)(800)$$
$$2800 = (4)(700)$$
$$3000 = (5)(600)$$

$$R = (3 + x)(800 - 100x)$$

Apr 20-5:50 PM

4. Determine the number which exceeds the square of the same number by the greatest possible amount.

Apr 20-5:51 PM

Many word problems dealing with quadratic relations in factored form are concerned with financial situations (i.e., money). You may find the following definitions useful:

Revenue: The income for the business; the amount of money that comes into the business; positive.

Cost: The expenses for the business; the amount of money that goes out of the business; negative.

Profit: The difference between *revenue* and *cost*.
        Profit = Revenue – Cost.
A positive profit is good for a business, and a negative profit (also called a *loss*) is bad.

Break-Even Point : The point where profit is zero. This is where profit changes between positive and negative.

Nov 3-11:03 PM

Assigned Work: stevesweeney.pbworks.com

        p. 147 # 12, 13, 14
        p. 157 # 13, 14, 15

Unit Test - Tuesday, November 8th

Suggested Review:

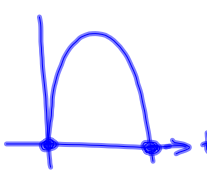   - read through all notes
        - revisit homework questions
            - redo questions that caused problems

        p.184 # 1, 3, 5, 6, 8, 9, 10, 11ace, 12
                14ace, 15ac, 16

        p.240 # 2, 6, 7, 9, 10, 11, 13ace, 16ace, 17ace
                19ace

12.

$$h = 20t - 5t^2$$

$$h = 5t(4-t)$$

Set $h = 0$

$$0 = 5t(4-t)$$

$5t = 0$ or $4 - t = 0$

$$\boxed{t = 0} \qquad \boxed{4 = t}$$

↑ hits ground

$$t_{vertex} = \frac{0+4}{2}$$

$$= 2$$

for h_{vertex}, sub $t_v = 2$

$$h_v = 5(2)(4-2)$$

$$= 10(2)$$

$$= 20 \qquad \therefore V(2, 20)$$

Nov 7-10:33 AM

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p.148

14. $y = 500 - 5x^2$

height ↙ ↘ time

raft dropped at $x = 0$

$$y = 500 - 5(0)^2$$

$$= 500$$

∴ helicopter is 500m high

(b) want zeroes $(y = 0)$

$$0 = 500 - 5x^2$$

$$0 = 5(100 - x^2)$$

$$0 = 5(10-x)(10+x)$$

$10 - x = 0$ or $10 + x = 0$

$$\boxed{10 = x} \qquad \boxed{x = -10}$$

inadmissable
inad
but $x > 0$
$t > 0$

∴ raft hits water after 10 seconds.

(c) set $x = 6$

$$y = 5(100 - 6^2)$$

$$= 5(100 - 36)$$

$$= 5(64)$$

$$= 320$$

(d) $y = 100$, what is x?

$$100 = 500 - 5x^2$$

$$-500 \quad -500$$

$$\frac{-400}{-5} = \frac{-5x^2}{-5}$$

$$80 = x^2$$

$$x = \pm\sqrt{80} \;, \; x > 0$$

$$x = \sqrt{80}$$

$$x \doteq 8.94$$

∴ after 8.94s, height is 100m

Nov 7-10:44 AM

p.158
 #14

  #10   80
   11   75
   12   70

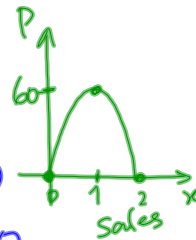$R = (10+x)(80-5x)$

Set $R = 0$

$0 = (10+x)(80-5x)$

$10+x=0$ or $80-5x=0$

Nov 7-10:58 AM

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p.147 #13

$P = 120x - 60x^2$

$P = -60x^2 + 120x$

$P = -60x(x-2)$

for zeroes, set $P=0$

$0 = -60x(x-2)$

$\dfrac{-60x}{-60} = \dfrac{0}{-60}$ $x-2=0$

$\boxed{x=0}$ $\boxed{x=2}$

axis of sym: $x = \dfrac{0+2}{2}$

               $= 1$

max value, sub $x=1$

$P = -60x(x-2)$

$P = -60(1)(1-2)$

$P = -60(-1)$

$P = 60$

∴ max profit is $\$60\ 000$

Nov 8-9:21 AM