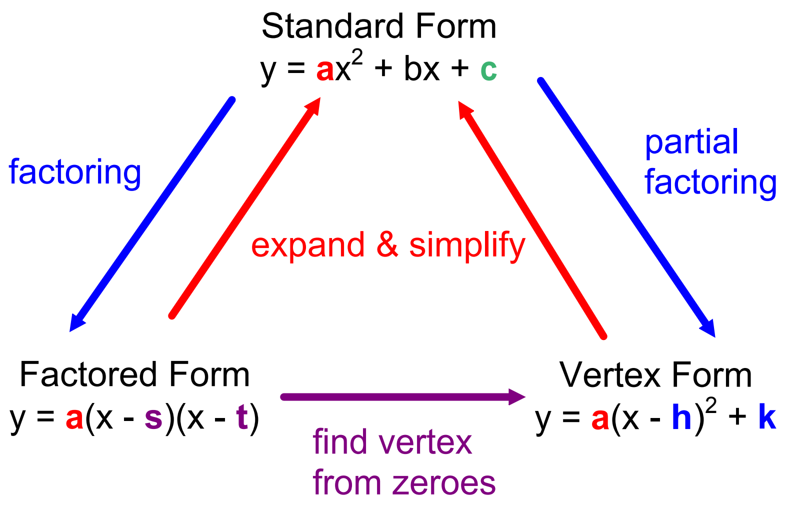
Relating Three Forms of a Quadratic Equation

Nov. 21/2011



Apr 12-2:18 PM

Ex.1 Expand & simplify each equation to obtain the standard form equation.

FOIL

(a) $y = 2(x + 5)(x - 1)$

$= 2(x^2 - x + 5x - 5)$

$= 2(x^2 + 4x - 5)$

$= 2x^2 + 8x - 10$

(b) $y = -0.5(x - 4)^2 + 3$

	x	-4
x	x^2	$-4x$
-4	$-4x$	16

$= -0.5(x-4)(x-4) + 3$

$= -0.5(x^2 - 4x - 4x + 16) + 3$

$= -0.5(x^2 - 8x + 16) + 3$

$= -0.5x^2 + 4x - 8 + 3$

$y = -0.5x^2 + 4x - 5$

Apr 12-2:18 PM

Ex.2 Write $y = x^2 - 4x + 3$ in
(a) factored form, and
(b) vertex form.

(a) $y = x^2 \underbrace{-4x}{} + 3$

S -4
P 3
I $-1, -3$

$y = \underbrace{x^2 - x}{} \underbrace{-3x + 3}{}$

$y = x\underbrace{(x-1)}_{a} - 3\underbrace{(x-1)}_{a}$

$y = xa - 3a$

$y = a(x-3)$

$y = (x-1)(x-3)$

(b) find zeroes, set $y=0$

$0 = (x-1)(x-3)$

$x-1=0$ or $x-3=0$

$x=1$ $\quad$ $x=3$

$x_{mp} = \frac{1+3}{2}$

$= 2 \rightarrow$ x-value of vertex (h)
$\rightarrow$ axis of symmetry

Sub $x=2$ into $y = x^2 - 4x + 3$

$y = (2)^2 - 4(2) + 3$

$y = 4 - 8 + 3$

$y = -1 \rightarrow$ y-value of vertex (k)
$\rightarrow$ optimum value

vertex $(2,-1)$, $a=1$ from standard form

$y = a(x-h)^2 + k$

$y = (1)(x-2)^2 + (-1)$

$y = (x-2)^2 - 1$

Apr 15-10:32 AM

Ex: Determine the vertex, and the vertex form, of
$y = x^2 - 12x + 5$

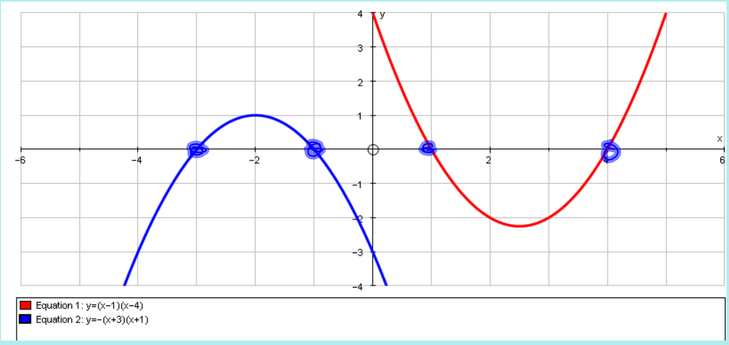
S -12
P 5
I $-1, -5$

cannot be factored

Apr 15-10:43 AM

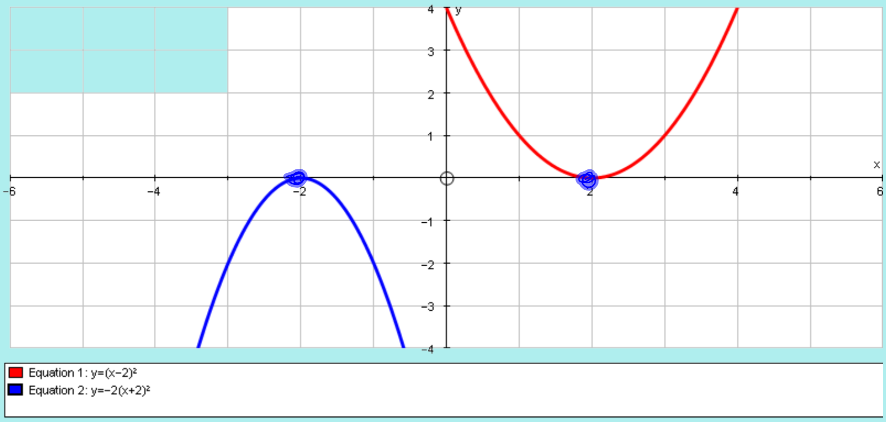
If the parabola crosses the x-axis, the x-coordinates of the crossing points are called the zeroes, or roots, or x-intercepts.

A parabola may have two zeros:



Apr 15-9:06 PM

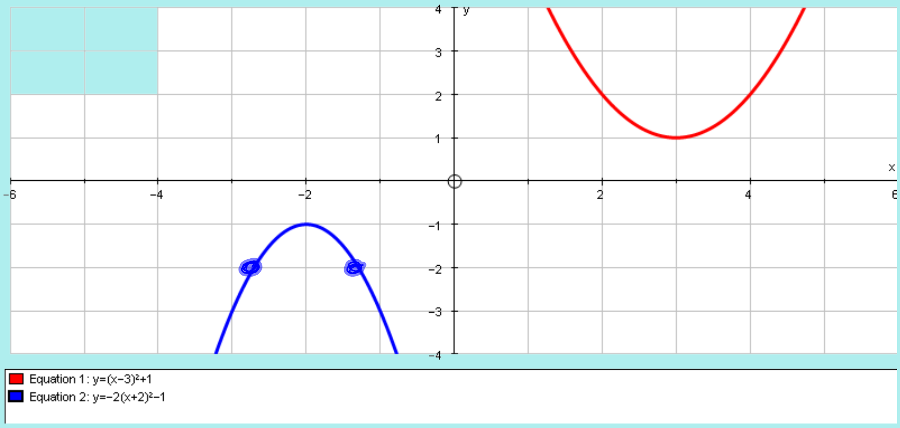
Or one zero:



$y = a(x-h)^2$

Apr 15-9:09 PM

Or no zeroes:

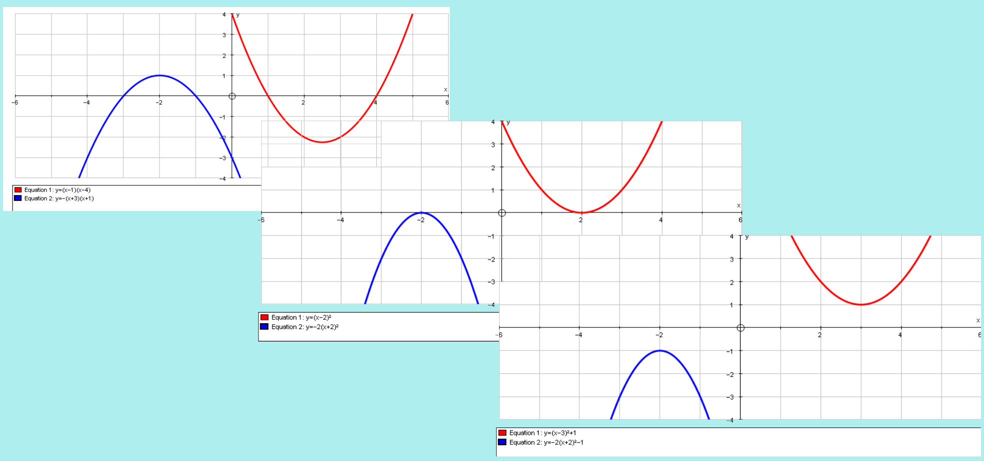


Apr 15-9:12 PM

Recall:

(1) Factored form indicates the <u>zeroes</u> of the quadratic relation.

(2) A quadratic relation can have 0, 1, or 2 zeroes.



Nov 20-8:17 PM

Not all quadratics have zeroes, which means they cannot be factored. Instead, use <u>symmetry</u> to perform a <u>partial factoring</u>.

1) Determine two points that have the same y-value.
   - start with a point that is given and then find the <u>matching point</u> with the same y-value
   - the y-intercept is usually a good choice

2) Find the x-value of the vertex (h) using symmetry

3) Find the y-value of the vertex (k) by subbing h into the original equation.

Apr 12-2:33 PM

Ex.3 Determine the vertex, and the vertex form, of
$y = x^2 - 12x + 5$

y-int $(0,5)$

S -12
P 5
I ___
cannot be factored

want the matching point where $y=5$

set $y=5$

$5 = x^2 - 12x + 5$
$-5 \qquad\qquad -5$

$0 = x^2 - 12x$

$0 = x(x-12)$

$x=0$ or $x-12=0$
$x=12$

$(0,5)$ y-int

$(12,5)$ matching point

$x_{mp} = \frac{0+12}{2}$

$= 6 \rightarrow$ x-value of vertex
$h=6$

Sub $x=6$ into $y=x^2-12x+5$

$y = (6)^2 - 12(6) + 5$

$y = 36 - 72 + 5$

$y = -31 \rightarrow$ y-value of vertex
$k=-31$

vertex $(6,-31)$, $a=1$ (from standard form equation)

$\boxed{y = (x-6)^2 - 31}$

Apr 12-2:42 PM

Ex. 4 Determine the vertex, and the vertex form, of

$y = -3x^2 + 15x + 2$

Apr 12-2:43 PM

Assigned Work:

p.293 # 4c, 5ac, 6ac, 9ac, 10ac → review

p.301 # 4, 5acef, 7ace → partial factoring

b

Apr 15-12:08 PM

p.293 #4(c)

$V(\underset{h}{-3},\underset{k}{2})$ $P(-1,14)$

$y=a(x-h)^2+k$

$y=a(x-(-3))^2+2$

$y=a(x+3)^2+2$

Sub $P(-1,14)$

$14=a(-1+3)^2+2$

$14=a(4)+2$

$-2 \qquad -2$

$\frac{12}{4}=\frac{4a}{4}$

$\boxed{a=3}$ $y=3(x+3)^2+2$

Nov 22-9:12 AM

p.293 #6(c)

need 5(c) $y=-(x-4)^2+4$

standard → expand & simplify

$y=-1(x-4)(x-4)+4$

$=-1(x^2-4x-4x+16)+4$

$=-1(x^2-8x+16)+4$

$=-x^2+8x-16+4$

$y=-x^2+8x-12$ S 8, P 12, I 2,6

$=-x^2+2x+6x-12$

$=-x\underbrace{(x-2)}_{a}+6\underbrace{(x-2)}_{a}$

$=-xa+6a$

$=a(-x+6)$ $-x+6$

$=(x-2)(-x+6)$ ✓ $=-1x+6$

$=(x-2)(-1)(x-6)$ $=-1(x-6)$

$=-(x-2)(x-6)$ *

$y=-x^2+8x-12$ S −8, P 12, I −2,−6

$=-1(x^2-8x+12)$

$=-1(x^2-2x-6x+12)$

$=-1(x(x-2)-6(x-2))$

$=-1(x-2)(x-6)$

Nov 22-9:15 AM

9(c)

$y = -(x+5)^2 + 1$

$= -(x+5)(x+5) + 1$

$= -(x^2 + 10x + 25) + 1$

$= -x^2 - 10x - 25 + 1$

$= -x^2 - 10x - 24$

$= -(x^2 + 10x + 24)$

$= -(x+4)(x+6)$

	x	$+5$
x	x^2	$5x$
$+5$	$5x$	25

Nov 22-9:25 AM

p. 301 #4(b)

$\underbrace{(3,0) \quad (7,0)}_{\text{zeroes}} \quad (9,-24)$

(a) $x_{mp} = \frac{3+7}{2}$

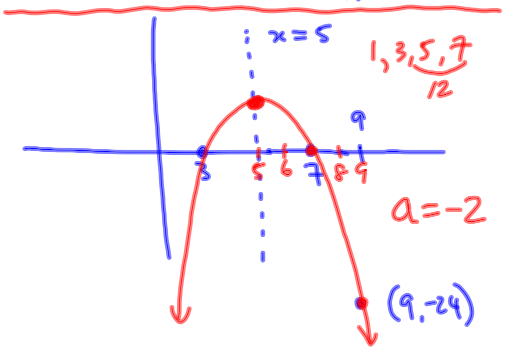
(c) $\boxed{x = 5} \rightarrow V(5, k)$

$y = a(x-5)^2 + k$

sub $(3,0)$ and $(9,-24)$

→ 2 equations, 2 unknowns

→ solve for a and k.



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p. 301 5(f)

$y = x^2 - 11x + \underline{21}$

$\downarrow$

y-int $(0, 21)$

set $y = 21$ to find matching point

$21 = x^2 - 11x + 21$
$-21 \qquad\qquad\quad -21$

$0 = x^2 - 11x$

$0 = x(x - 11)$

$x = 0$ or $x = 11$

$(0, 21) \qquad (11, 21)$

(ii) $x_{mp} = \frac{0 + 11}{2}$

$\boxed{x = \frac{11}{2}}$

$\boxed{x = 5.5}$

(iii) sub $x = 5.5$ into $y = x^2 - 11x + 21$

$y = (5.5)^2 - 11(5.5) + 21$

$= 30.25 - 60.5 + 21$

$= -9.25$

$V(5.5, -9.75)$

Nov 22-9:40 AM