Vertex Form by Completing the Square

Nov. 24/2011

Recall:

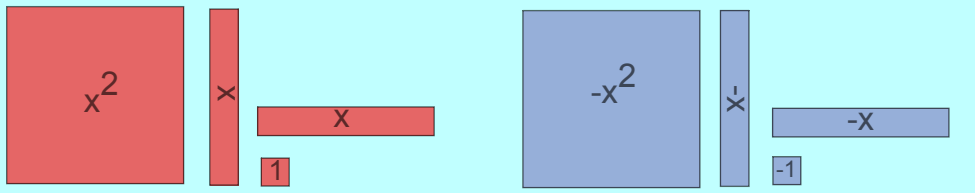
Vertex form: $y = a(x - h)^2 + k$

Note that $(x - h)^2$ is a perfect square.

In general, for perfect square trinomials,

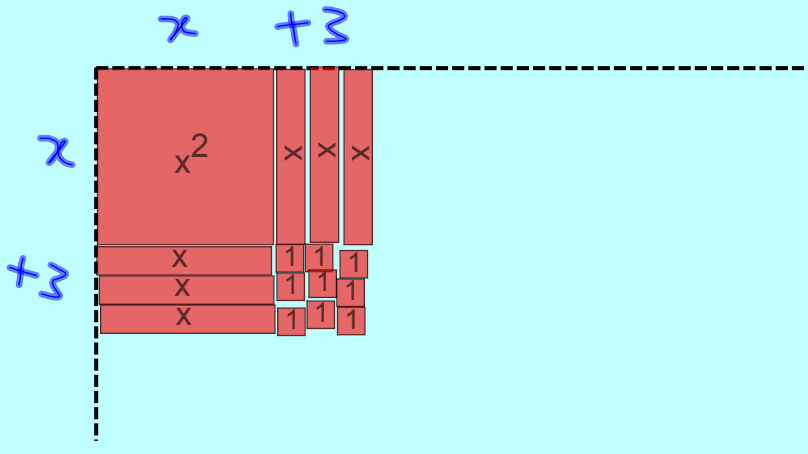
$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

Apr 12-2:42 PM

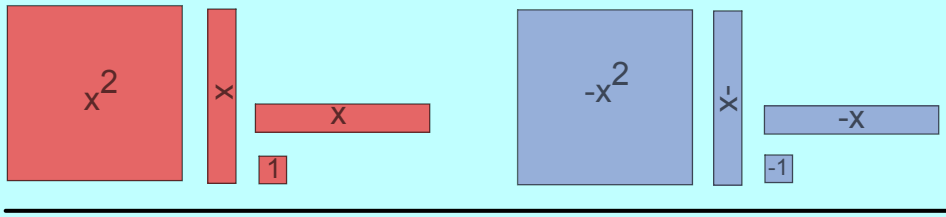


Identify the missing constant so that the trinomial is a perfect square trinomial, then factor it.

$x^2 + 6x + \underline{9} = (x+3)^2$

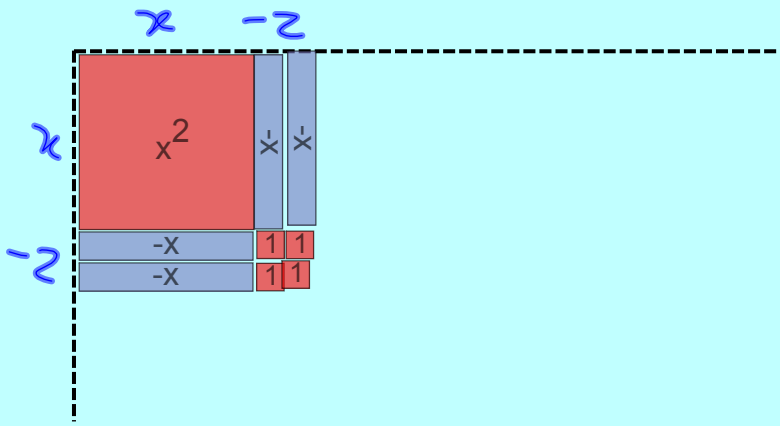


Mar 25-8:02 AM



Identify the missing constant so that the trinomial is a perfect square trinomial, then factor it.

$x^2 - 4x + \underline{4} = (x-2)^2$



Mar 25-8:02 AM

Ex.1 What is missing from these perfect squares?

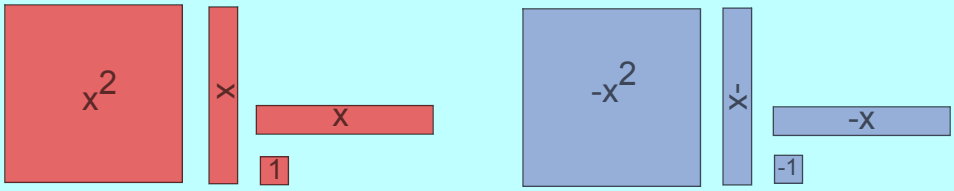
(a) $x^2 + 10x + \underline{25} = (x+5)^2$

$\frac{+10}{2} = +5 \qquad 5^2 = 25$

(b) $x^2 - 18x + \underline{81} = (x-9)^2$

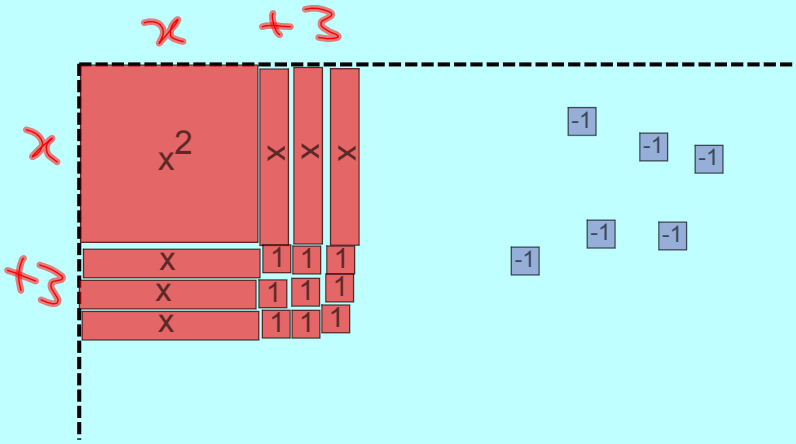
	x	-9
x	x^2	$-9x$
-9	$-9x$	$+81$

Nov 23-8:41 PM



Identify the missing constant so that the trinomial is a perfect square trinomial. You will have some tiles "left over".

$x^2 + 6x + 3 = (x+3)^2$ -6



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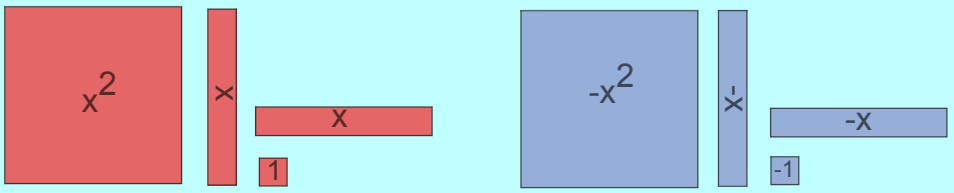
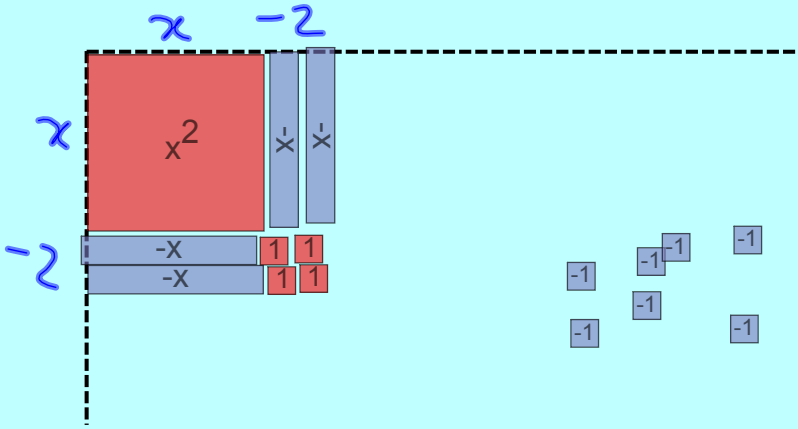


Identify the missing constant so that the trinomial is a perfect square trinomial. You will have some tiles "left over".

$x^2 - 4x - 3 = (x-2)^2$ -7



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Steps: if $a \neq 1$

1) Factor out 'a' from the first two terms. x^2 and x

2) Force a perfect square for the factored first two terms.
- take half of the 'b' term and square it

3) Collect the constants.

Ex.2 Complete the square for each of the following

a) $y = x^2 + 12x - 7$

$b = +12$

$\frac{+12}{2} = +6$

$(+6)^2 = 36$

$y = x^2 + 12x + 36 - 36 - 7$

$y = (x+6)^2 - 43$

May 3-7:51 PM

b) $y = x^2 - 20x + 15$

$y = (x-10)^2 - 85$

	x	-10
x	x^2	$-10x$
-10	$-10x$	$+100$

$+15$

-100

Nov 23-9:00 PM

c) $y = 3x^2 + 12x + 11$

$y = 3(x^2+4x) + 11$

x^2+4x
$= x^2+4x+4-4$
$= (x+2)^2 - 4$

	x	$+2$
x	x^2	$2x$
$+2$	$2x$	$+4$

-4

$y = 3[(x+2)^2 - 4] + 11$

a

$y = 3[a-4] + 11$

$y = 3a - 12 + 11$

$y = 3(x+2)^2 - 1$

May 4-8:48 AM

d) $y = -x^2 + 6x + 13$

$y = -1(x^2-6x) + 13$

$y = -[x^2-6x+9-9] + 13$

$y = -[(x-3)^2 - 9] + 13$

$\frac{-6}{2} = -3$

$(-3)^2 = 9$

$y = -(x-3)^2 + 9 + 13$

$y = -(x-3)^2 + 22$

Nov 23-9:00 PM

(e) $y = -2x^2 + 7x - 3$

$y = -2\left(x^2 - \frac{7}{2}x\right) - 3$

$y = -2\left[x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{49}{16}\right] - 3$

$y = -2\left[\left(x - \frac{7}{4}\right)^2 - \frac{49}{16}\right] - 3$

$y = -2\left(x - \frac{7}{4}\right)^2 + \frac{2(49)}{16} - 3$

$y = -2\left(x - \frac{7}{4}\right)^2 + \frac{49}{8} - \frac{24}{8}$

$y = -2\left(x - \frac{7}{4}\right)^2 + \frac{25}{8}$

$y = -2(x - 1.75)^2 + 3.125$

$\frac{-7}{2} \div 2$

$= -\frac{7}{2} \times \frac{1}{2}$

$= -\frac{7}{4}$

$\left(-\frac{7}{4}\right)^2 = \frac{49}{16}$

Nov 25-9:55 AM

Assigned Work:

7ab

p. 331 # 2, 3, 4, 5ace, 7bde, 9, 11, 16

Take up any word problems

Mar 20 - 4:57 PM

5(a)

$y = x^2 + 14x$

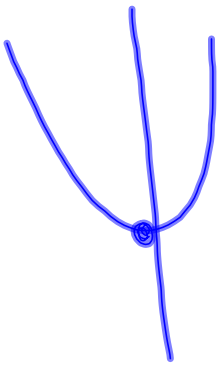
$y = x^2 + 14x + 49 - 49$

$y = (x+7)^2 - 49$

$b = +14$

$\frac{+14}{2} = +7$

$(+7)^2 = 49$



$V(-7, -49)$

$\therefore$ min value is -49

Nov 25-9:11 AM

16. $y = x^2 - 2x - 35$ $\frac{-2}{2} = -1$, $(-1)^2 = 1$

① complete square

$y = x^2 - 2x + 1 - 1 - 35$

$y = (x-1)^2 - 36$

$V(1, -36)$

② factored form

$y = x^2 - 2x - 35$ $S = -2$, $P = -35$, $7, -5$

$y = x^2 - 7x + 5x - 35$

$y = x(x-7) + 5(x-7)$

$y = (x-7)(x+5)$

zeroes: Set $y = 0$

$0 = (x-7)(x+5)$

$x - 7 = 0$ $x = 7$ $\quad x + 5 = 0$ $x = -5$

$x_{MP} = \frac{7 + (-5)}{2}$

$= 1$

Sub $x = 1$ into $y = (x-7)(x+5)$

$y = (1-7)(1+5)$

$y = -36$

$V(1, -36)$

③ partial factoring

$y = x^2 - 2x - 35$

find y-int

set $y = -35$

$\frac{-35}{+35} = x^2 - 2x - \frac{35}{+35}$

$0 = x^2 - 2x$

$0 = x(x-2)$

$x = 0$ or $x - 2 = 0$, $x = 2$

$(0, -35)$ $\quad (2, -35)$

$x_m = \frac{0+2}{2}$

$= 1$

Sub $x = 1 \longrightarrow y = -36$

Nov 25-9:14 AM

Assigned Work:

p.331 # 2, 3, 4, 5ace, 7bde, 9, 11, 16

Some review

p.335 # 3, 4, 5, 7, 10

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p.331 #4

$y = -2x^2 + 12x - 11$

$y = -2[x^2 - 6x] - 11$ $\quad \frac{-6}{2} = -3$, $(-3)^2 = 9$

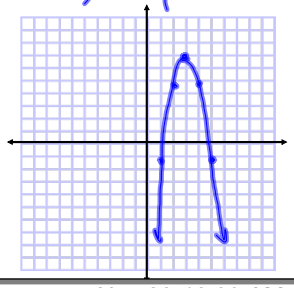
$y = -2[x^2 - 6x + 9 - 9] - 11$ $\quad (x-3)^2$

$y = -2[(x-3)^2 - 9] - 11$

$y = -2(x-3)^2 + 18 - 11$

$y = -2(x-3)^2 + 7$ $\quad 1, 3, 5, 7, \ldots$ $\quad -2, -6, -10, \ldots$

$V(3, 7)$ opens down



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5(e)

$y = -4.9x^2 - 19.6x + 0.5$

$y = -4.9\left[x^2 + 4x\right] + 0.5$

$y = -4.9\left[x^2 + 4x + 4 - 4\right] + 0.5$

$y = -4.9\left[(x+2)^2 - 4\right] + 0.5$

$y = -4.9(x+2)^2 + 19.6 + 0.5$

$y = -4.9(x+2)^2 + 20.1$

$V(-2, 20.1)$ max is 20.1

Nov 28-10:45 AM

9.

$P = -15x^2 + 240x - 640$

$P = -15\left[x^2 - 16x\right] - 640$

$P = -15\left[x^2 - 16x + 64 - 64\right] - 640$

$P = -15\left[(x-8)^2 - 64\right] - 640$

$P = -15(x-8)^2 + 960 - 640$

$P = -15(x-8)^2 + 320$

$V(8, 320)$

$\therefore$ have 8 customers to max profit

Nov 28-10:49 AM